502/E1

Third order convergence of Improved Newton's method for systems of nonlinear equations

H P S Nishani[1], S Weerakoon[1], T G I Fernando[2] and L M Liyanage[1]
[1]*Department of Mathematics, Faculty of Applied Sciences, University of Sri Jayewardenepura, Nugegoda*
[2]*Department of Statistics and Computer Science, Faculty of Applied Sciences, University of Sri Jayewardenepura, Nugegoda*

Improved Newton's Method (INM) is a widely accepted third order iterative method introduced in the late 90's to solve nonlinear equations. It has become so popular among numerical analysts that it records more than 460 citations in recognized international journals. However, even after more than a decade of the initial introduction of INM, none have taken the challenge of extending the INM for systems of nonlinear equations. We have previously shown with the help of computational evidence that the order of convergence is still three. However, until now it has not been established scientifically by providing a proof for the third order convergence.

In this paper, we establish the third order convergence of the Improved Newton's method given by (1) & (2), to solve a system of non-linear equations $\underline{F}(\underline{x}) = \underline{0}$, by providing a complete proofwith the help of some matrix algebra, the Taylor series expansion and the Binomial expansion.

$$\underline{X}_{n+1} = \underline{X}_n - 2\left[J(\underline{X}_n) + J\left(\underline{X}^{\lambda}{}_{n+1}\right)\right]^{-1} F(\underline{X}_n) \Longrightarrow (1)$$

$$\text{where} \underline{X}^{\lambda}{}_{n+1} = \underline{X}_n - \left[J(\underline{X}_n)\right]^{-1} F(\underline{X}_n) \Longrightarrow (2)$$

Keywords:Convergence, order of convergence, Newton's method, iterative methods

menakaliyanage@yahoo.com Tel:0714478812