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Improved Newton's method for functions of two variables

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Improved Newton's Method (INM) is an internationally accepted third order iterative method introduced in the late 90's to solve nonlinear equations. It has become so popular among numerical analysts that it records more than 380 citations in recognized international journals. This paper extends the INM to functions of two variables by applying the INM to the system:

$$f(x,y) = 0$$
$$g(x,y) = 0$$

using the following iterative equations:

$$(x_{n+1} - x_n)[f_x(x_n) + f_x(x^*_{n+1})] + (y_{n+1} - y_n)\left[f_y(x_n) + f_y(x^*_{n+1})\right] = -2f(x_n)$$
$$(x_{n+1} - x_n)[g_x(x_n) + g_x(x^*_{n+1})] + (y_{n+1} - y_n)\left[g_y(x_n) + g_y(x^*_{n+1})\right] = -2g(x_n)$$

where x^*_{n+1} is the $(n+1)^{th}$ iterate obtained by applying the Newton's method.
We also show computationally that the method we derived is still third order convergent by applying it to several representative functions.

The purpose of this publication is to let the researchers know of this very efficient method for root finding of functions of several variables. By applying the established formulae, researchers and people in industries would be able to reach the root much faster when facing problems involving functions of two variables. Further, interested and capable researchers will be encouraged by the results provided, to try and find the proof of the third order convergence of the Improved Newton method for functions of two or more variables thus paving the way for hundreds of researchers worldwide to come up with similar methods for functions of several variables, just like the way our publication for one variable case inspired over 300 researchers all over the world, demonstrating the power of sharing knowledge.

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