



831/E1

Invertibility of Modified Buffon Transformation matrices

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The Buffon transformation deals with polygons. It generates an infinite polygon sequence by connecting the midpoints of edges of each subsequent polygon. Hence it generates polygonal designs by repeating this transformation. Buffon transformation on an n -gon can be represented by a $n \times n$ matrix and when we consider the modified Buffon transformation obtained by choosing a point which divides each edge of the polygon into $a : 1 - a$; $a < 1$, we get the following matrix as the corresponding transformation matrix T_n^a .

$$T_n^a = \begin{bmatrix} 1 & a & a & 0 & 0 & \dots & 0 & 0 \\ 0 & 1-a & a & 0 & 0 & \dots & 0 & 0 \\ 0 & 0 & 1-a & a & 0 & \dots & 0 & 0 \\ \vdots & \vdots & & & & \ddots & \vdots & \vdots \\ & & & & & & 0 & 1-a & a \\ a & 0 & 0 & 0 & \dots & 0 & 0 & 1-a \end{bmatrix}_{n \times n}$$

Here we prove by induction that T_n^a is invertible in all other cases except when the order of T_n^a is even and $a = 1/2$. Subsequently we prove that the set of modified Buffon transformation matrices form a group when n is odd.