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Convexifiable lagrangian and sufficient optimality

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Consider the following mathematical programming problem with bounds on variables:

$$\text{(P)} \quad \underset{x \in \mathbf{R}^n}{\text{Minimize}} \quad f_0(x)$$

$$\text{subject to } f_j(x) \le 0, \quad j = 1, 2, \ldots, m,$$

$$u_i \le x_i \le v_i, \quad i = 1, 2, \ldots, n$$

where $u_i < v_i$, *for* $i = 1, 2, \ldots, n$ and $f_j : \mathbf{R}^n \to \mathbf{R}$, $j = 0, 1, 2, \ldots, m$ are continuously differentiable functions.

Recently a class of non-convex programming problems called "the convexifiable programming problem" was introduced which is of the form (P) where each function f_j, $j = 0, 1, 2, \ldots, m$ becomes convex under certain domain and range monotone transformations. Precisely, the programming problem (P) is called a (strictly) convexifiable programming problem if for each $j = 0, 1, 2, \ldots, m$ there exist $T_j : \mathbf{R} \to \mathbf{R}$ and $t : \mathbf{R}^n \to \mathbf{R}^n$ such that $T_j \circ f_j \circ t$ is (strictly) convex where t is separable, strictly monotone and continuously differentiable and T_j is strictly increasing and differentiable. It was established that the most desired property of convex (or generalized convex) programming problems, that "local minimizers are indeed the global minimizers" holds for convexifiable programming as well.

In this paper as an extension of the above result, the following KKT sufficiency is established. Whenever the Lagrangian function of (P),

$L(x, \lambda) = f_0(x) + \sum_{j=1}^{m} \lambda_j f_j(x)$, $x \in \mathbf{R}^n$, $\lambda \in \mathbf{R}^m$ is convexifiable, each Karush-Kuhn-Tucker point of (P) indeed becomes the global minimizer of (P). It is worth while noting that convexifiability of the Lagrangian function does not require convexifiability of the objective function and constraints as required by the convexifiable programming problem.

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