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Physically meaningful zeros and poles of elastic S-matrix element

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It is found that an essential singularity is introduced at the origin of the complex k-plane in the S-matrix element in addition to the infinite number of zeros and poles apparently introduced due to the Coulomb potential. It can be shown that the essential singularity at the origin is a mathematical artifice and hence is unphysical. Necessary conditions for poles corresponding to decaying resonance states, capture states and closed states can also be derived using standard mathematical techniques in the presence of the Coulomb potential.

The partial wave radial wave equation of angular momentum l corresponding to elastic scattering is given by,

$$\left[\frac{d^2}{dr^2} + k^2 - \frac{l(l+1)}{r^2} \right] u_l(k, r) = \frac{2\mu}{\hbar^2} [V(r) + V_c(r) + iw(r)] u_l(k, r)$$

where $V(r)$ is the real part of the potential containing spin – orbit potential and volume term, $w(r)$ the imaginary part of the optical potential, $V_c(r)$ the Coulomb potential and k is the incident wave number. The S-matrix element $S_l^n(k)$ is now can be written as

$$S_l^n(k) = (-1)^l \frac{\Gamma(l+1-i\eta)}{\Gamma(l+1+i\eta)} \frac{W'_{i\eta, l+\frac{1}{2}}(2ikr) - P_l(k, r) W_{i\eta, l+\frac{1}{2}}(2ikr)}{W'_{-i\eta, l+\frac{1}{2}}(-2ikr) - P_l(-k, r) W_{-i\eta, l+\frac{1}{2}}(-2ikr)}$$

where $P_l(k, r) = \frac{u'_l(k, r)}{u_l(k, r)}$ and $r \geq R_m$, cutting off the potential tails at R_m . It is clear that there are infinite number of zeros and poles of the S-matrix element due to the fact that $\eta = \frac{\mu z_1 z_2 e^2}{\hbar^2 k}$ and the structure of the Gamma function. We have found that all salient features of physically meaningful zeros and poles of $S_l^n(k)$ can be derived from the above functional form of $S_l^n(k)$.