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Underestimators and duality in mathematical programming

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In this paper, the following mathematical programming problem is studied.

(P) Minimize $f_o(x)$

subject to $x \in X_0, f_j(x) \leq 0, \ j = 1,2,\ldots,m,$

where X_0 is an open subset of $\Re^n$ and $f_j : X_0 \to \Re$, $j = 0,1,2,\ldots,m,$ are continuously differentiable functions and $D = \{x \in X_0 / f_j(x) \leq 0, j = 1,2,\ldots,m\}$, feasible set.

The objective of this study is to present how the duality properties of the underestimator approximation problems (UP) (defined below) is related to the corresponding properties of the original problem (P).

We begin with underestimator. The function $h : \Re^n \to \Re$ is an under estimator of a function $f : \Re^n \to \Re$ at $\bar{x}$ over $C \in \Re^n$, if for each $x \in C, f(x) \geq h(x)$ and $f(\bar{x}) = h(\bar{x})$.

Let $\bar{x}$ be a feasible point of (P). Suppose that for each $j = 0,1,..,m$, f_j admits an underestimator $g_j(\bar{x},.)$ at $\bar{x}$ over X_0 (note that such underestimators always exist as the function f_j is underestimator of itself). Now define the following approximation problem (UP) involving underestimators of the objective function and the constraints of the original problem (P).

Underestimator approximation problem is defined by

(UP) Minimize $g_0(\bar{x}, y)$

subject to $y \in X_0, g_j(\bar{x}, y) \leq 0, \ j = 1,2,\ldots,m.$

It is first presented that Karush-Kuhn-Tucker (KKT) points of the model problem (P) are indeed KKT points of an underestimator approximation problem (UP). This property is essential to see that minimizers of (P) are indeed feasible points of the corresponding dual underestimator problems. It is also mainly proved that weak/strong duality relationship between underestimator approximation problem and its dual imply that weak/strong results hold for the original problem (P). Thus, new duality results can well be derived for non-convex optimization problems with several local solutions that are not global via underestimator approximation problems. A new duality result is furnished to illustrate this claim.

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