SECTION E$_1$

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Exact formula for the sum of the squares of the Bessel and the Neumann function of the half-odd integer order

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Sum of the squares of the spherical Bessel function and the Neumann function of the same order of an integer has been found to be very useful in theoretical nuclear physics. This sum can be obtained from the corresponding sum of the half-odd integer Bessel and Neumann functions. To our surprise, there is no exact formula for the afore mentioned sum but an approximate formula is available, which has been obtained by G.N. Watson, and is valid for the complex argument whose real part is greater than zero, and the absolute value of the upper bound of the error term is undefined in case of half-odd integers. The same formula has been obtained by G.N. Watson, which is valid for all complex arguments, using the sophisticated mathematical method called Barnes' method. However, the error term in this formula is very difficult to estimate. We have shown that the Watson formula is exact, in the important case of positive half-odd integers, using elementary mathematics and the Nicholson formula. Watson formula can be written as $J_\nu^2(z)+N_\nu^2(z)\approx\frac{2}{\pi z}\sum_{k=0}^{\infty}\frac{(2k-1)!!\Gamma\left(\nu+k+\frac{1}{2}\right)}{2^k z^{2k}k!\Gamma\left(\nu-k+\frac{1}{2}\right)}$ and the upper bound of the error term R_p is given by $\left|R_p\right|<\left|\frac{\cos\nu\pi}{\cos R(\nu\pi)}\right|\frac{p!\left|(R(\nu),p)\right|}{(2p)!}2^{2p}\sinh^{2p}t$ in approximating the ratio $R=\frac{\cosh 2\nu t}{\cosh t}$ is undefined in the case of ν equal to half-odd integer, where J_ν , N_ν are the Bessel function of the first kind and the second kind respectively and $R(\nu)$ is the real part of ν .When $\nu=n+\frac{1}{2}$, $(R(\nu),p)$ is given by $p!(R(\nu),p)=\frac{\Gamma(n+1+p)}{\Gamma(n+1-p)}$ .We have justified our result by using complex contour integration and showing that R_p is exactly zero in this important case.