White Noise in a stock market model

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Market is a place where buyers and sellers meet for day-to-day transactions and for long-term transactions. Stock market is a place where shares are sold and bought and it is a special kind of a financial market where market share price is determined at every moment. Share price index measures share price fluctuations on a daily basis. It is a well-known scenario that sudden disturbances in the society have a great impact on the price of goods and stocks in financial markets. The aim of this study was to investigate the effect of White Noise on the time path of the price function of linear stock market models via a Wiener process (inclusion of random disturbances in the form of differentials).

Under the assumption that the growth rate of the stock price $P(t)$ at time *t* is a linear superposition of a time independent parameter μ and the speed of the Wiener-process $W(t)$ with a time dependent White Noise Multiplier $\sigma(t)$, the governing equation of the stock market model takes the form,

$\frac{1}{P}\frac{dP}{dt} = \mu + \sigma\frac{dW}{dt}$, where $P(0) = P_0 > 0$ and $t \in [0,T]$.

This is a Stochastic- Initial Value Problem (S-IVP) and using chain rule in stochastic calculus the following stochastic differential form can be established, where $\sigma \in C[0,T]$:

$$d(\log_e P) = (\mu - \frac{1}{2}\sigma^2)\, dt + \sigma\, dW$$

Solving this we get the time path of the price function $P(t)$. Upper bounds for expectation $E[P(t)]$ and variance $V[P(t)]$ are obtained using well-known CBS and GRB inequalities. That is, we have established the following results:

(i) $P(t) = P_0 \exp[\mu t - \frac{1}{2}\int_0^t \sigma^2 ds + \int_0^t \sigma\, dW]$.

This shows that White Noise has a significant impact on the time path of the Price function.

(ii) $E(P(t)) = P_0 \exp[\int_0^t \mu(s)\, ds] = P_0 e^{\mu t} \le P_0 e^{\mu T}$.

Thus the expectation of the time path of the price function is free from the white noise.

(iii) $V(P(t)) \le \|P^2\|_2 \|\sigma^2\|_2 \exp\left(\int_0^t \mu^2\, ds\right) = \|P^2\|_2 \|\sigma^2\|_2\, e^{\mu^2 T}$. Moreover this can be utilised to detect the strength of the White Noise $\|\sigma\|_S$ of the market model over a time period $[0,T]$.

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