Lie group analysis of nonlinear ordinary differential equations

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In this paper, the Lie group theoretical method of integration of second and third-order nonlinear ordinary differential equations (NLODEs) is presented. Point transformations that form a continuous Lie group of transformations or Lie point symmetries which leave the equations form invariant are studied.

The infinitesimal generator $X = \xi(x,y)\partial_x + \eta(x,y)\partial_y$ of an nth-order NLODE, $y^{(n)} = H\left(x, y, \ldots, y^{(n-1)}\right)$, $n \geq 1$, is a Lie point symmetry generator if $X^{[n]}\left(y^{(n)} - H\right)\Big|_{y^{(n)}-H=0} = 0$,

where, $X^{[n]}$ is the nth-prolongation of X.

The knowledge of the Lie point symmetries admitted by the equations combined with the notion of solvability of the Lie algebra is used in the integration process. The consecutive reduction of the order of the equations is done by the ideals of the Lie algebra.

For the second-order NLODE, an exact solution was obtained and the third-order equation was integrated by quadratures.

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