# Products of pre-connected spaces

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A subset $K$ of a topological space $(X,\tau)$ is said to be pre-open if there exists a $U \in \tau$ such that $K \subseteq U \subseteq Cl_X K$. It is easy to see that every open set is pre-open; but, in general the converse is not true. The complement of a pre-open set is called a pre-closed set. The space $X$ is said to be pre-connected if it is not the union of two non-empty disjoint pre-open sets. The aim of this study is to investigate the necessary and sufficient conditions needed on factor spaces to ensure the productivity of pre-connectedness in arbitrary products with respect to Tychonoff topology.

In this direction, as the machinery we establish the following results:

Lemma: The continuous image of a pre-connected set is pre-connected.

Remark: Pre-connectedness is a topological invariant.

Lemma: Let $A$ be a pre-connected subspace of a topological space $(X,\tau)$. If, $A \subseteq B \subseteq Cl_X A$ then $B$ is pre-connected.

Lemma: The $\aleph_0$- weak-topological sums are dense subspaces of the Tychonoff product space $\prod_{i \in I} X_i$.

Lemma: Product of two pre-connected spaces is pre-connected.

Corollary: The property pre-connectedness is preserved by finite products $\prod_{i=1}^{n} X_i$.

Utilizing the above results we conclude the following:

Theorem: Arbitrary Tychonoff product space **Error! Objects cannot be created from editing field codes.**is pre-connected if and only if each factor space **Error! Objects cannot be created from editing field codes.** is pre-connected.