E1 230

Some Algebraic Properties of Stochastic Polygons

The definition of a right stochastic polygon was introduced by Skornyakov. By analogy with ordinary polygons and using probability techniques, some definitions such as a right stochastic subpolygon, a stochastic factor polygon, a free stochastic polygon, a stochastic indecomposable polygon, can be introduced. Some basic algebraic properties of right stochastic polygons as objects of the category of the right stochastic polygons are derived in terms of isomorphism, epimorphism, coproduct and decomposability.

Theorem A free right stochastic S-polygon F with a basis X, where S is a monoid, and $X \subseteq F$, can be represented as a disjunctive union of right stochastic polygons $x_i\ S$, where $x_i \in X$, and each such stochastic polygon is isomorphic to the right stochastic S- polygon S, such that $S'_S (S_1, S_2) = \delta_S (S_1S', S_2)$ for any $S', S_1, S_2 \in S$

On the set of stochastic maps $SF(X, X \times S)$ we introduce the operation *, such that

$$(f_1 * f_2)(x_1, x_2 s_1) = \sum \sum f1 (x_1, xs)\ f_2 (x, x_2 s')\ \delta_s (s's, s_1)$$

Then the monoid of endomorphisms of the free right stochastic S- polygon F with the basis X is isomorphic to the monoid $SF^*(X, X \times S)$ with the introduced operation * .

The analogous results for ordinary (nonstochastic) polygons can be found in the work of Kilp and Michalev (1986). Any stochastic polygon corresponds to the stochastic automation without exit. The results obtained may be useful for solving decomposition problems of stochastic polygons and automata.