

A location dispersion test for ordered categorical data in $r \times k$ tables

In the statistical literature for testing differences of two or several distributions in ordinal $2 \times k$ tables or $r \times k$ tables, we normally use Pearson's chi-squared test (1900), Wilcoxon test (1945), Mantel's test (1963) and Nair's test (1986). In those tests, Pearson's chi-squared test does not have good power against ordered alternatives and the Wilcoxon test or Mantel's test is designed for testing location differences and Nair's test is designed to detect dispersion alternatives.

Jayasekara and Yanagawa (Bulletin of informatics and Cybernetics, 31(1999), 180-190) have proposed a statistical test called Q_t test which has higher powers than Pearson's chi-squared test and Nair's location and dispersion and dispersion test for non-linear responses in $2 \times k$ ordered categorical tables. In this paper, we extended the above Q_t test for detecting non-linear differences of several populations in $r \times k$ ordered categorical tables.

The proposed test is a location-dispersion test which is sensitive for both location shift and scale shift which seem common, for example, in clinical trial data. So the test

statistic is expressed as functions of the Wilcoxon test and test whose score is orthonormal to the Wilcoxon score. Gram-Schmidt orthonormalization procedure was applied to find orthonormal scores. The test statistic gives equal weights to the first component that detects location alternatives and to the second component that detects dispersion alternatives.

The asymptotic distribution of the proposed test statistic are obtained both under the null and alternative hypothesis. Considering several distributions belong to linear, parabolic shapes and several other non-linear shapes, the powers of the test are computed and compared with Mantel's test and Pearson's Chi-squared test. According to the results of the simulation, the proposed test has higher powers than Mantel's test and Karl Person's Chi-squared test if the pattern of the response is linear or parabolic shape.