E1-32: Approximating roots of unknown functions of two variables

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Finite Difference Newton's method (FDN) was introduced (1996) and it was extended to functions of two variables to approximate roots of simultaneous nonlinear equations in the absence of partial derivatives. The proposed scheme replaces partial derivatives of the functions in Newton's method by appropriately chosen forward or backward difference formulae. The order of convergence of the proposed method was shown to be at least two and computational results overwhelmingly support the theory.

In the present effort, we have tried to approximate the roots of two simultaneous nonlinear equations

$\{f_1(x,y)=0;\ f_2(x,y)=0\}$

using only the values of the functions in a rectangular domain

$\{D=(x_i,y_i)\ a \le x_i \le b\ ;\ c \le y_j \le d,\ i = 0,1,\ldots, n\ ;\ j = 0,1,\ldots, m\}$.

Even though we are unaware of the closed forms of the above functions, to apply FDN method we need the values of those functions at arbitrary points. Thus we used bicubic Lagrange surface patch interpolation method to approximate the functions within each of the square grid (each grid contains 16 node points) and applied the FDN method to the approximated functions.

We applied this procedure to several sets of data generated by various types of (known) functions and we also applied the FDN method using the closed forms of those functions with the same initial guesses and compared the results. Results obtained for the surface interpolation and for the actual functions were very close, suggesting the validity of bicubic interpolant approach. Hence we don't have to hesitate applying this procedure to discrete sets of data without knowing the closed forms of the functions.