E1-31: Third order convergence of Improved Newton's method for roots of nonlinear equations

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Newton's method given by :

$$x^*_{n+1} = x^*_n - f(x^*_n)/ f(x^*_n)$$

generates the sequence $\{ x^*_n \}$ which is expected to converge to the root α of a nonlinear equation, converges to the root quadratically. As a result, after some iterations, the process doubles the number of correct decimal places or significant digits at each iteration.

We introduced the iterative scheme:

$$x_{n+1} = x_n - 2f(x_n)/[f'(x_n) + f'(x^*_{n+1})]; n=0,1,2,..$$

Where $x^*_{n+1} - x_n = f(x_n)/ f'(x_n)$

improving Newton's Method (1997). In that paper we showed that it was second order convergent and the computed results overwhelmingly suggest that the proposed method should be at least third order convergent.

After trying along many directions we were successful in proving that it is in fact third order convergent and in the present paper we give that proof. We have

convergence of the Improved Newton's Method is established beyond any doubt researchers who need a faster convergent method can make use of it.