

E1-16: An Analysis of direct resistance heating of a circular sheet blank using moving electrodes

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This paper presents results of a numerical study on direct resistance heating of a circular metal sheet of uniform thickness.

The power balance equation:

$$\frac{\partial}{\partial r} \left(\lambda \frac{\partial T}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \theta} \left(\lambda \frac{\partial T}{\partial \theta} \right) + F(r, \theta) = \frac{\epsilon \sigma}{b} \left(T^4 - T_o^4 \right) + \frac{h_o}{b} (T - T_o) + dC \frac{\partial T}{\partial t}$$

which describes the temperature (T) of the circular plate is obtained. The rate of heat generated $F(r, \theta)$, and current density is related to each other. The current

density (J) of a uniform sheet satisfies the continuity equation

$$\frac{\nabla J}{\nabla} + \frac{\partial V}{\partial t} = 0$$

Assuming that the electric charge density does not depend on time explicitly, this continuity equation can be written as

$$\frac{\partial^2 \phi}{\partial r^2} + \frac{1}{r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi}{\partial \theta^2} - \frac{\alpha}{1 + \alpha(T - T_o)} \left(\frac{\partial T}{\partial r} \frac{\partial \phi}{\partial r} + \frac{1}{r^2} \frac{\partial T}{\partial \theta} \frac{\partial \phi}{\partial \theta} \right) = 0$$

Here we assume the relationships

$$\frac{J}{\rho} = -\frac{1}{\rho} \nabla \phi \text{ and } \rho = \left[\rho_1 (1 + \alpha(T - T_o)) \right]$$

where ϕ is electrical potential and ρ is electrical resistivity.

We use the Finite Difference Techniques to solve these equations simultaneously. Initially, we obtain numerical results of the above equations when a circular sheet blank is heated by passing an electrical current using 2 electrodes pressed to the sheet at the ends of one diameter. The results show that temperature distribution of the sheet is not uniform enough.

To obtain a more uniform temperature distribution, electrical current is passed through several diameters in a staggered pattern and we find the numerical solution of the above equations. These results show that if the boundary is maintained at room temperature, the final temperature distribution is fairly uniform except for a small ring near the boundary.

Notation:

T = temperature, λ = thermal conductivity, ϵ = total emissivity, σ = Stefan Boltzmann constant, b = thickness of the plate, h_o = convective heat transfer coefficient, d = mass density, and C = specific heat capacity