E1-22 Improved Newton method for finding roots of a nonlinear equation

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Newton's method that approximates the root of a non-linear equation in one variable using the value of the function and its derivative, in an iterative fashion, is probably the best known and most widely used algorithm and it converges to the root quadratically.

In this paper, we suggest an improvement to the iteration of Newton's method. Derivation of the Newton's method involves an indefinite integral of the derivative of the function and the relevant area is approximated by a rectangle. In the proposed scheme we approximate this indefinite integral by a trapezium instead of a rectangle, thereby reducing the error in the approximation.

It is shown that the suggested method converges quadratically and computational results overwhelmingly support this theory. Even though we have shown that new method converges quadratically, in several cases, average order of convergence exceeds three. For most of the functions we tested, the average order of convergence in the Newton's method is less than two and for the proposed method, it always exceeds two.