

E1-21 Approximating roots of unknown functions

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Finite Difference Newton's method (FDN) was introduced by Weerakoon (1996) to approximate single roots of non-linear equations in the absence of the derivative. The proposed scheme replaces the derivative of the function in Newton's method by appropriately chosen forward to backward difference formulae. The order of convergence of the proposed method is shown to be at least two and computational results overwhelmingly supported the theory.

In the present effort, we have tried to approximate the roots of an unknown function using only a set of discrete values $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ available. In the process we need to approximate the function so that the values of the function are available as required by the computational procedure. We used natural cubic spline interpolation to approximate the function within each subinterval and applied both Newton's method and the FDN method using the approximated function. We applied this procedure to several sets of data generated by various types of (known) functions to see how it works and the results were very encouraging.

We also applied both Newton's method and the FDN method to actual functions with the same initial guess to compare the results. Results obtained for the spline interpolant and for the actual function were very close. Further, when the initial guess is somewhat away from the root, FDN method seems to converge much faster, suggesting that in the circumstances FDN is the best method to use. Hence we don't have to hesitate applying this procedure to a discrete set of data without knowing the closed form of the function.