E1-02 Finite difference Newton's method for functions of two variables

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Researchers frequently encounter systems of nonlinear equations as a result of modelling many physical processes. Newton's interactive method is the most widely used numerical scheme to find roots of equations. This method has major drawbacks: (1) Necessity to differentiate the functions (which sometimes could be cumbersome) and feed them into the computer; (2) Inability to implement the algorithms when there are no closed forms for the functions except for a discrete set of function values; (3) Necessity to evaluate partial derivatives for the Jacobian matrix at each iterative step.

To overcome these difficulties, Finite Difference Newton's method was introduced by Weerakoon (1994) for nonlinear equations of one variable. The objective here is to extend it to two-dimensions by replacing the 2 x 2 Jacobian matrix of the Newton's method by appropriate finite difference approximations, without slowing down the process of convergence. The approach was the same in approximating derivatives but since the derivatives of functions of two or more variables involve Jacobian matrices, difficulties were faced when approximating four derivatives even in the case of two-dimensions.

The results obtained were very encouraging. The method was applied to several functions, such as polynomials, trigonometric and exponential functions and in all those cases, suggested methods converged as fast as Newton's method and sometimes even faster.