

SUMUDU TRANSFORM - A NEW INTEGRAL
TRANSFORMATION TO SOLVE DIFFERENTIAL
EQUATIONS AND CONTROL
ENGINEERING PROBLEMS

G.K. Watugala

Dept. of Mech. Engineering, University of Moratuwa

There are numerous integral transforms (1 - 4) to solve differential equation problems encountered in engineering. Of these Laplace transformation is widely used in solving differential equations and control engineering problems. The essence of Laplace transformation is the mapping of time-domain (t-domain) function $f(t)$ to a Laplace-domain (s-domain) function $F(s)$ by an integral transformation. By this the differentiation and integration operations in t-domain is made equivalent to multiplication and division by s in the s-domain. The variable s and the transformed function $F(s)$ in the s-domain are treated as dummies in this process (5) and their physical significance is irrelevant.

It is possible to solve differential equations and control engineering problems by a transformation in which the differentiation and integration of a function $f(t)$ in t-domain is made equivalent to division and multiplication of transformed function $F(u)$ in a u-domain. The new transformation (which is called Sumudu transform for the time being) possesses many interesting properties which make the visualization of the process easier. Some of the properties of Sumudu transform are: (1) the unit-step function in t-domain is transformed to unity in u-domain, (2) scaling of the function $f(t)$ in t-domain is equivalent to the scaling of $F(u)$ in the u-domain by the same scale factor. (3) The function $F(u)$ is even, if and only if $f(t)$ is even, and the same holds for odd functions. (4) The limit of $f(t)$ as t tends to zero is same as the limit of $F(u)$ as u tends to zero. (5) For several cases the limit of $f(t)$ as t tends to infinity is same as the limit of $F(u)$ as u tends to infinity. (6) The slope of the function $f(t)$ at $t=0$ is same as the slope of $F(u)$ at $u=0$. Hence u and $F(u)$ are no longer dummies but can be treated as replicas of t and

$f(t)$. It is even possible to express u and $F(u)$ in the same units of t and $f(t)$ respectively.

In this paper, the definition, properties and the application of Sumudu transform is given. Finally it is shown that Sumudu transform is useful in many instances as an introduction to transforms for students who are planning to be practising engineers. For the engineering graduates planning to engage in higher studies, an understanding of Sumudu-transform would help as a stepping stone in mastering Laplace transform.

References:

1. Wolf, K.B. (1979) Integral transforms in science and engineering, NY, Plenum Press.
2. Miles, J.W., (1971) "Integral transform in applied mathematics", Cambridge, Cambridge University Press.
3. Handbook of Applicable Mathematics, Vol IV, Chapter 13 Integral Transforms, Ed. by I.N. Sneddon, A Wiley-Interscience Publication.
4. Integral transform and their applications, Applied mathematical sciences series, v. 25, Springer Verlag, 2nd ed.
5. Automatic Control Engineering by Raven, F.H., McGraw-Hill Kogakusha Ltd, International Student Edition, p.122.