Sri Lankan Journal of Applied Statistics, Vol.9, 2008, p.74-83

Construction and Selection of Conditional Double Sampling Plan Indexed Through Six Sigma Quality Levels

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ABSTRACT

Six Sigma is a concept, a process, a measurement, a tool, a quality philosophy, a culture and a management strategy adopted for the improvement in the system of an organization, which when applied reduce the number of defects to 3.4 or below per million opportunities. The main aim of six sigma is to reduce costs/wastages as much as possible and improve the quality of the product to the maximum satisfaction of the consumer. Focusing on reduction of defects will result in enhanced quality with more satisfaction for the consumer and high profit for the producer. In this paper a new procedure for the construction and selection of Conditional Double Sampling Plan (CDSP) indexed through Six Sigma Quality Level-1 (SSQL-1) and Six Sigma Qutlity Level-2 (SSQL-2) are presented. Tables are constructed and presented for the easy selection of the plans.

Key words: Six Sigma Quality Level, Operating Characteristic curve, Poisson Distribution, Double Sampling Plan, Conditional Double Sampling Plan, Intervened Random effect Poisson Distribution.

INTRODUCTION

Baker and Brobst (1978) proposed Conditional Double Sampling procedures and these sampling procedures have operating characteristic (OC) curves identical to those of comparable double sampling procedures. Conditional double sampling is operationally different from double sampling in that the results of the second sample, if required, are obtained from a related lot and not from the current lot. Vijayaraghavan (1990) has provided procedures and tables for the selection of conditional double sampling plans for various entry parameters. Suresh and Ramkumar (1996) introduced the concept of MAAOQ in the construction of single sampling plan. Radhakrishnan (2002) constructed the conditional double sampling plan indexed with Maximum Allowable Average Outgoing Quality (MAAOQ). Sampathkumar (2007) constructed mixed sampling plan with conditional double sampling plan as a reference plan using various entry parameters. Sekkizhar (2007) constructed conditional double sampling plan indexed through MAPD and MAAOQ using Intervened Random effect Poisson Distribution (IRPD) as the basic distribution. Radhakrishnan and Sivakumaran (2008) constructed single sampling plan indexed through Six Sigma Quality Level (SSQL).

In this paper a Conditional Double Sampling Plan is constructed by assuming the probability of acceptance of the lot, $P_a(p)$ as $1\text{-}3.4 \times 10^{-6}$, the concept of six sigma quality suggested by Motorola (1980). The proportion defective corresponding to this probability in the OC curve is termed as Six Sigma Quality Level-1 (SSQL-1= p_{ss1}). This new sampling plan is constructed with a point on the OC curve (p_{ss1}, $1\text{-}\alpha_1$), where α_1=3.4 x 10^{-6}. Dodge and Roming (1942) constructed sampling plans using (AQL, $1\text{-}\alpha$) and (LQL, β) with $\alpha = 0.5$ and $\beta = 0.10$. The proportion defective corresponding to the probability $2\alpha_1$ in the OC curve is termed as Six Sigma Quality Level-2 (SSQL-2= p_{ss2}). Sampling plans can also be constructed with a point on the OC curve (p_{ss2}, β_1), where $\beta_1=2\alpha_1$. The tables are also provided for the easy selection of the plans using the above concepts.

OPERATING PROCEDURE

In Conditional double sampling plan by attributes the lot acceptance procedure is characterized by the parameters N, n_1, n_2, c_1, c_2 and c_3. The operating procedure of a Conditional Double Sampling Plan is given below.

Step 1: Select a random sample of size n ($=n_1=n_2$) from a lot of size 'N'

Step 2: Inspect all the articles included in the sample. Let 'd_1' be the number of defectives in the sample.

Step 3: If $d_1 \leq c_1$, accept the lot.

Step 4: If $d_1 > c_3$, reject the lot.

Step 5: If $c_1+1 \leq d_1 \leq c_3$, take a second sample of size n_2 from the remaining lot and count the number of defectives 'd_2'.

Step 6: If $d_2 \leq c_2$ or $d_1+d_2 \leq c_3$ accept the lot, otherwise reject the lot.

CONDITIONS FOR APPLICATION

- Production is steady, so that results of past, present and future lots are broadly indicative of a continuing process.

- Lots are submitted sequentially in the order of their production.
- Inspection is by attributes, with the lot quality defined as the proportion defective.
- Human involvement should be less in the manufacturing process.
- The plans are more useful to the companies in the developed and developing countries, which adopt Six Sigma quality initiatives in their process.

GLOSSARY OF SYMBOLS

The symbols used in this paper are as follows:

N - Lot size

n_1 - First sample size

n_2 - Second sample size

c_1 - First sample acceptance number

c_2 - Second sample acceptance number

c_3 - Third acceptance number

d_1 - Number of defective items counted in the first sample

d_2 - Number of defective items counted in the second sample

AQL - Acceptable Quality Level

LQL - Limiting Quality Level

MAPD - Maximum Allowable Percent Defective

MAAOQ - Maximum Allowable Average Outgoing Quality

p_{ss1} - Six sigma quality level-1

p_{ss2} - Six sigma quality level-2

α - Producer's risk

β - Consumer's risk

α_1 - Modified Producer's risk

β_1 - $2\alpha_1$ (Modified Consumer's risk)

σ_1 - Sigma level of the process when the first sample is taken

σ_2 - Sigma level of the process when the second sample is taken

OC FUNCTION

Under Poisson model, the OC function of the Conditional Double Sampling Plan is given by

$$Pa(p) = \sum_{i=0}^{C_1} p_i + p_{C_1+1} \sum_{i=0}^{C_3-C_1+1} q_i + \ldots\ldots + p_{C_3} \sum_{i=0}^{C_3-C_2} q_i \qquad (1)$$

where $p_i = \dfrac{e^{-n_1 p}(n_1 p)^i}{i!}$, $q_i = \dfrac{e^{-n_2 p}(n_2 p)^i}{i!}$

CONSTRUCTION OF CONDITIONAL DOUBLE SAMPLING PLANS FOR A SPECIFIED P_{SS1}

By fixing the probability of acceptance of the lot, $P_a(p)$ as $1\text{-}3.4 \times 10^{-6}$ with Poisson Distribution as the basic distribution, the values of $n_1 p_{ss1}$ are obtained for various combinations of c_1, c_2 and c_3 using a Visual Basic program and presented in Table 1. The sample size 'n_1' is obtained using $n_1 = n_1 p_{ss1}/p_{ss1}$ and the values of n_2 (where $n_2 = n_1$), c_1, c_2 and c_3 (where $c_3 = c_2+1$) are also obtained for various combinations of p_{ss1}.

The sigma-1(σ_1) and sigma-2 (σ_2) values are calculated using the Process Sigma Calculator (http://www.isixsigma.com/).

Example 1

For a given p_{ss1} = 0.00007 and c_1=3, c_2=4 and c_3=5, the value of $n_1 p_{ss1}$ is selected from Table 1 as 0.2030 and the corresponding sample size 'n_1' is computed as n_1=0.2030/0.00007= 2900 and $n_2 = n_1$, which are associated with σ_1=4.6 and σ_2=4.7 sigma levels. Hence the parameters of CDSP are n_1=2900, n_2=2900, c_1=3, c_2=4 and c_3=5 for a specified six sigma quality level, p_{ss1} = 0.00007.

Explanation:

If the manufacturer of cell phone batteries fixes the quality as p_{ss1} = 0.00007(7 defective batteries out of 1 lakh cell phone batteries), then take a sample of 2900 cell phone batteries from the manufactured lot of a particular week or month and count the number of defective batteries (d_1). If $d_1 \leq 3$ accept the lot of batteries manufactured in that week or month and if $d_1 > 5$, reject the lot of batteries manufactured in that week or month and improve the quality of the same. If $4 \leq d_1 \leq 5$, take a second sample of 2900 batteries from the remaining lot and count the number of defective batteries (d_2). If $d_2 \leq 4$ (or) $d_1+d_2 \leq 5$ accept the lot of batteries manufactured in the same week or month, otherwise repeat the procedure for the other manufactured lots in the week or month to accept or reject the quality of the batteries.

Table 1: Parameters of CDSP for a specified P_{ss1}

C_1	C_2	C_3	n_1p_{ss1}
0	1	2	0.0030
0	2	3	0.0270
0	3	4	0.0890
1	2	3	0.0280
1	3	4	0.0910
1	4	4	0.1140
2	2	4	0.0280
2	3	4	0.0940
2	4	4	0.1260
2	4	5	0.1900
3	4	5	0.2030
3	4	6	0.2170
3	5	6	0.3210
3	6	6	0.3350
4	5	6	0.3150
4	6	6	0.3770
4	6	7	0.4820
5	6	6	0.4560
5	6	7	0.5340
5	7	7	0.5550
6	7	8	0.7490
6	8	9	0.8860
7	8	9	0.9960
8	9	10	1.2720

Example 2

For a given p_{ss1} = 0.00004, c_1=4, c_2=6 and c_3=7, the value of $n_1 p_{ss1}$ is selected from Table 1 as 0.4820 and the corresponding sample size 'n_1' is computed as n_1=0.4820/0.00004=12050 and n_2=n_1, which are associated with σ_1=4.9 and σ_2=5.0 sigma levels. Hence the parameters of CDSP are n_1=12050, n_2=12050, c_1=4, c_2=6 and c_3=7 for a specified six sigma quality level, p_{ss1} = 0.00004.

Explanation

In a big Courier service organisation, if the management fixes the quality level as p_{ss1} = 0.00004 (4 defective services out of 1 lakh deliveries in a week or month), then take a sample of 12050 deliveries from the deliveries of a week or month and count the number of defective deliveries (d_1). If $d_1 \leq 4$ accept the lot quality of service delivered in that week or month and if $d_1 > 7$ reject that lot of deliveries in the same week or month and improve the quality of the service. If $5 \leq d_1 \leq 7$, take a second sample of 12050 deliveries from the remaining week or month and count the number of defective deliveries (d_2). If $d_2 \leq 6$ (or) $d_1+d_2 \leq 7$, accept the quality of deliveries provided in the same week or month, otherwise repeat the procedure on the other weeks or months deliveries to accept or reject the quality of service.

The OC curves of the plans provided in Example 1 and Example 2 are presented in the Figure 1.

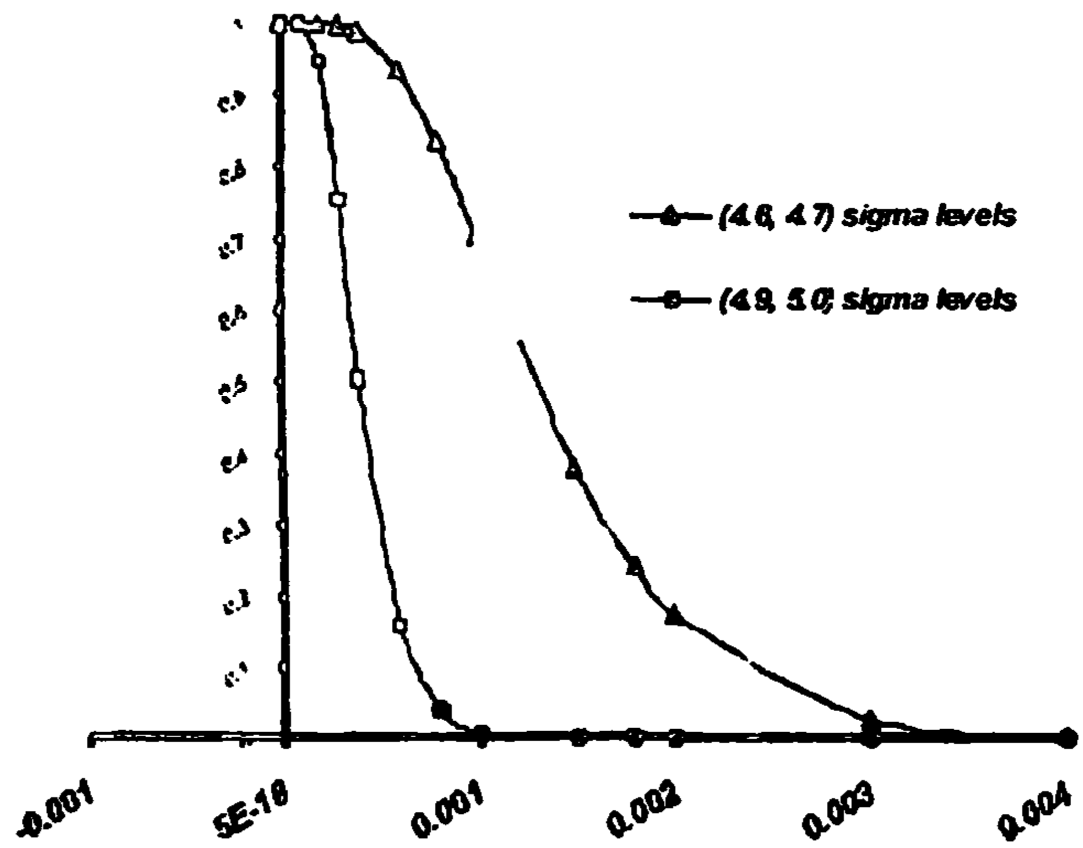

Figure 1: Comparison of OC curves for various sigma levels Indexed through p_{ss1}

CONSTRUCTION OF CONDITIONAL DOUBLE SAMPLING PLANS FOR A SPECIFIED P_{SS2}

By fixing the probability of acceptance of the lot, $P_a(p)$ as $\beta_1 = 2\alpha_1$ where $\alpha_1=3.4 \times 10^{-6}$ with Poisson Distribution as the basic distribution, the values of n_1p_{ss2} are obtained for various combinations of c_1, c_2 and c_3 using a Visual Basic program and are presented in Table 2. The sample size 'n_1' is obtained using $n_1 = n_1p_{ss2}/p_{ss2}$ and the values of n_2 (where $n_2 = n_1$), c_1, c_2 and c_3 (where $c_3 = c_2+1$) are also obtained for various combinations of p_{ss2}.

The sigma-1(σ_1) and sigma-2(σ_2) values are calculated using the Process Sigma Calculator (http://www.isixsigma.com/).

Table 2: Parameters of CDSP for a Specified P_{SS2}

C_1	C_2	C_3	n_1p_{ss2}
0	1	2	11.8999
0	2	3	11.9120
0	3	4	11.9840
1	2	2	14.6499
1	2	3	14.6499
1	3	4	14.6499
1	4	4	14.6499
2	2	4	16.9880
2	3	4	16.9880
2	4	4	16.9880
2	4	5	16.9880
3	4	5	19.1200
3	4	6	19.1200
3	5	6	19.1200
3	6	6	19.1200
4	5	6	21.1200
4	6	6	21.1200
4	6	7	21.1200
5	6	6	23.0260
5	6	7	23.0260
5	7	7	23.0260
6	7	8	24.8610
6	8	9	24.8610

Example 3

For a fixed p_{ss2} = 0.006, c_1=3, c_2=4 and c_3=5, the value of n_1p_{ss2} is selected from Table 2 as 19.1200 and the corresponding sample size 'n_1' is computed as n_1=19.1200/0.006= 3187 and n_2=n_1 which are associated with σ_1=4.6 and σ_2=4.7 sigma levels. Hence the parameters of CDSP are n_1=3187, n_2=3187, c_1=3, c_2=4 and c_3=5 for a specified six sigma quality level, p_{ss2} = 0.006.

Explanation

In the big Hospital/Health care service organisations, if the patients fix the quality of service as p_{ss2} = 0.006 for a satisfied service (6 dissatisfied patients out of 1 thousand patients), then take a sample of 3187 in or outpatients reported for a week and count the number of dissatisfied patients (d_1). If $d_1 \leq 3$ accept the quality of services provided in the same week and if $d_1 > 5$ reject the quality of service and suggest for improvement. If $4 \leq d_1 \leq 5$, take a second sample of 3187 patients from the remaining week and count the number of dissatisfied patients (d_2). If $d_2 \leq 4$ (or) $d_1+d_2 \leq 5$ accept the quality of service provided in the particular week, otherwise repeat the same procedure to accept or reject the quality of service provided by the organisation.

Example 4

For a fixed p_{ss2}=0.002, c_1=3, c_2=4 and c_3=5, the value of n_1p_{ss2} is selected from Table 2 as 19.1200 and the corresponding sample size 'n_1' is computed as n_1=19.1200/0.002= 9560 and n_2=n_1 which are associated with σ_1=4.9 and σ_2=5.0 sigma levels. Hence the parameters of CDSP are n_1=9560, n_2=9560, c_1=3, c_2=4 and c_3=5 for a specified six sigma quality level, p_{ss2} = 0.002.

Explanation

In a Transport providing organisation (Railway or Bus service), if the passengers fix the quality of service as p_{ss2} = 0.002 for a satisfied service (2 faulty services out of 1 thousand), then take a sample of 9560 passengers on a particular day or week and count the number of dissatisfied passengers (d_1). If $d_1 \leq 3$ accept the quality of service provided during that day or week and if $d_1 > 5$, reject the quality of service provided in that day or week and suggest for improvement. If $4 \leq d_1 \leq 5$, take a second sample of 9560 passengers from the remaining service of the day or week and count the number of dissatisfied passengers (d_2). If $d_2 \leq 4$ (or) $d_1+d_2 \leq 5$ accept the quality of service provided , otherwise repeat the same procedure for the other groups of passengers selected during the day or week.

The OC curves of the plans provided in Example 3 and Example 4 are presented in the Figure 2.

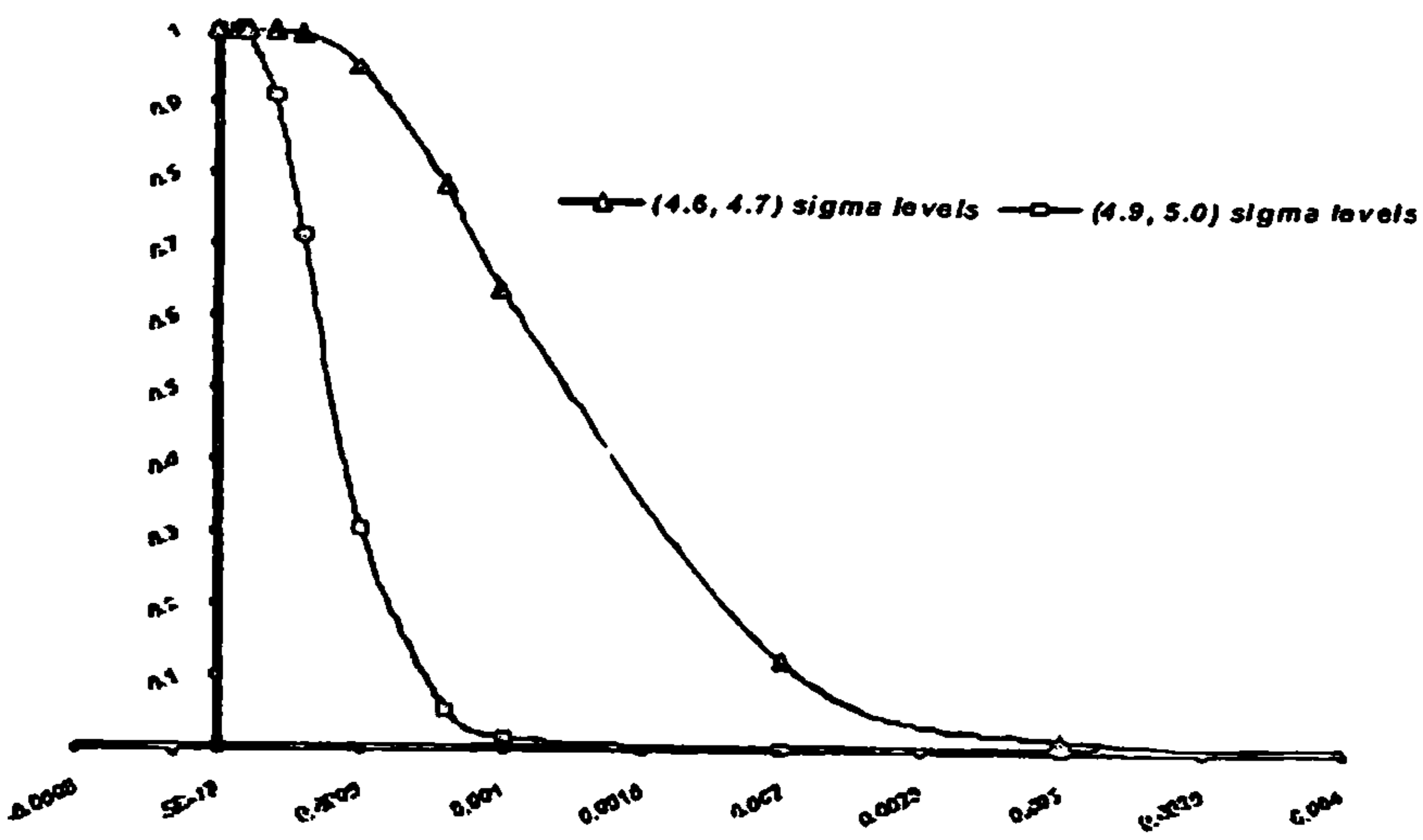

Figure 2: Comparison of OC curves for various sigma levels Indexed through p_{ss2}

CONCLUSION

In this paper a new procedure for the selection of Conditional Double Sampling Plan indexed through Six Sigma Quality Level-1 (p_{ss1}) and Six Sigma Quality Level-2 (p_{ss2}) are presented separately. These plans are very effective in place of classical plans indexed through AQL or LQL. These plans are very useful for the companies using six sigma quality initiatives in the manufacturing process. These plans are also more suitable for the companies in the developed and developing countries, which adopts six sigma quality initiatives in their manufacturing process. The tables are also provided for easy selection of the plans. These plans are very useful to the companies because the producer's risk and consumer's risk are less, which results in more satisfaction for the consumers and higher profit for the manufacturers.

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