A Review of Block Designs with Neighbour Effects

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ABSTRACT

Experiments in agriculture, horticulture, and forestry often show a neighbour effect that is the response on a given plot is affected by the treatments on neighbouring plot as well as by the treatment applied to that plot. When treatments are varieties, neighbour effects may be caused by differences in height, root vigor, or germination date, especially on small plots. As a result, the estimate of treatment differences may deviate because of this interference from neighbouring units. Neighbour-balanced designs ensure that no treatment is unduly disadvantaged by its neighbours. The allocation of treatments in these designs is such that every treatment occurs equally often with every other treatment as neighbours. Some aspects of block designs with neighbour effects have been presented considering the designs in linear blocks. Some methods of obtaining neighbour balanced block designs that are balanced and partially variance balanced for estimating the contrast pertaining to direct and neighbour effects have been described.

Keywords: Circular design, Neighbour balanced design, Neighbour effects, Partially variance balanced, Variance balanced

INTRODUCTION

Experiments conducted in agriculture often show neighbour effects i.e., the response on a given plot is affected by the treatments on the neighbouring plots as well as by the treatment applied to that particular plot. When treatments are varieties, neighbour effects may be caused by differences in height, root vigor, or germination date, especially on small plots, which are used in plant breeding experiments. Treatments such as fertilizer, irrigation, or pesticide may spread to adjacent plots causing neighbour effects. Such experiments exhibit neighbour effects, because the effect of having no treatment as a neighbour is different from the neighbour effects of any treatment. Competition or interference between neighbouring units in field

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experiments can contribute to variability in experimental results and lead to substantial losses in efficiency.

In case of block design setup if each block is a single line of plots and blocks are well separated, extra parameters are needed for the effect of left and right neighbours. An alternative is to have border plots on both ends of every block. Each border plot receives an experimental treatment, but it is not used for measuring the response variable. These border plots do not add too much to the cost of one-dimensional experiments.

Neighbour balanced designs, wherein the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for these situations and permit the estimation of direct and neighbour effects of treatments. The neighbour effects are also called as interference effects, indirect effects or remote treatment effects in the literature. Understanding the structure of these effects helps in minimizing the bias in treatments to great extent.

For v treatments, neighbour balance implies that the number of replication is divisible by v-1. An experimenter will rarely have the resources for 2(v-1) or more replications, so more attention is to be given to designs with replication exactly v-1.

Some aspects of neighbour balanced designs have been presented here considering the designs in linear blocks. Some methods of obtaining neighbour balanced block designs that are balanced and partially variance balanced for estimating the contrast pertaining to direct and left and right neighbour effects have been described.

BLOCK MODELS WITH NEIGHBOUR EFFECTS

The designs considered here are assumed to be in linear blocks, with neighbour effects only in the direction of the blocks (say left-neighbour or right-neighbour or both). Because the effect of having no treatment differs from the neighbour effects of any treatment, designs with border plots have been considered, which is, designs with one point added at each end of each block. The border plots receive treatments but are not used for measuring the response variables. The plots, which are not on the borders, are inner plots. The length of a block is the number of its inner plots. It is further assumed that all the designs are **circular**, that is the treatment on border plots is same as the treatment on the inner plot at the other end of the block.

There are different types of models related to neighbour effects,

(M-1) $Y_{ij} = \mu + \tau_{(i,j)} + \delta_{(i-1,j)} + \beta_j + \varepsilon_{ij}$

(M-2) $Y_{ij} = \mu + \tau_{(i,j)} + \delta_{(i-1,j)} + \alpha_{(i,j)(i-1,j)} \beta_j + \varepsilon_{ij}$

(M-3) $Y_{ij} = \mu + \tau_{(i,j)} + \delta_{(i-1,j)} + \delta_{(j+1,j)} + \beta_j + \varepsilon_{ij}$

(M-4) $Y_{ij} = \mu + \tau_{(i,j)} + \delta_{(i-1,j)} + \rho_{(j+1,j)} + \beta_j + \varepsilon_{ij}$

Where Y_{ij} is the response from the i^{th} plot in the j^{th} block ($i = 1, 2, .., k, j = 1, 2, \ldots, b$); μ is the general mean; $\tau_{(i,j)}$ is the direct effect of the treatment in the i^{th} plot of j^{th} block; β_j is the effect of the j^{th} block; $\delta_{(i-1,j)}$ is the left neighbour effect due to the treatment in the $(i-1)^{th}$ plot of j^{th} block; $\rho_{(j+1,j)}$ is the right neighbour effect due to the treatment in $(i+1)^{th}$ plot in j^{th} block; ε_{ij} are error terms independently and normally distributed with mean zero and variance σ^2.

M-1 represents the model with one sided neighbour effect (say left). For example, in varietal experiments tall varieties may shade the plot on the other side. In pesticides or fungicides experiments, part of the treatment may spread to the plot immediately downwind; so may spores from untreated plots. This model can also be adopted to temporal problems with carry-over effects. M-2 represents the model with interaction term between direct and neighbour effect.

M-3 corresponds to the model with undifferentiated neighbour effects from both left and right neighbours and M-4 represents block model with differentiated left and right neighbour effects.

Sometimes the aim of the experiment is to find a single treatment which can be recommended for use on larger spatial areas or over longer time periods than those used for individual treatments in the experiment: for example, a single variety of wheat to be grown in whole fields, or a single type of feed to be given to cows throughout a whole lactation. When the chosen treatment is in use, its only neighbor will be itself. Thus the effect of most importance is the sum of the direct effect of the treatment and the neighbor effect(s) of the same treatment. Bailey and Druilet, (2004) studied neighbour balanced designs for total effects. The vector of total effects for model with one sided neighbor effects (M-1) is defined by $\phi_1 = \tau + \delta$, the vector of total effects for model with interaction term (M-2) is defined by $\phi_2 = \tau + \delta$

$+ \alpha$, the vector of total effects for model with same additive effect from both sides (M-3) is defined by $\phi_3 = \tau + 2\delta$ and the vector of total effects for model with different additive effect from both sides (M-4) is defined by $\phi_4 = \tau + \delta + \rho$.

Model M-4 is now being considered for further description. Under the block design set-up with v treatments in b blocks of size k each, the information matrix for estimating the direct effects, left and right neighbour effects jointly is obtained as follows:

$$\mathbf{C}=\begin{bmatrix} \mathbf{R}_\tau - \frac{1}{k}\mathbf{N}_1\mathbf{N}_1' & \mathbf{M}_1 - \frac{1}{k}\mathbf{N}_1\mathbf{N}_2' & \mathbf{M}_2 - \frac{1}{k}\mathbf{N}_1\mathbf{N}_3' \\ \mathbf{M}_1' - \frac{1}{k}\mathbf{N}_2\mathbf{N}_1' & \mathbf{R}_\delta - \frac{1}{k}\mathbf{N}_2\mathbf{N}_2' & \mathbf{M}_3 - \frac{1}{k}\mathbf{N}_2\mathbf{N}_3' \\ \mathbf{M}_2' - \frac{1}{k}\mathbf{N}_3\mathbf{N}_1' & \mathbf{M}_3' - \frac{1}{k}\mathbf{N}_3\mathbf{N}_2' & \mathbf{R}_\rho - \frac{1}{k}\mathbf{N}_3\mathbf{N}_3' \end{bmatrix}, \quad (1)$$

where $\mathbf{M}_1$ $\mathbf{M}_2$ and $\mathbf{M}_3$ are the v × v incidence matrices pertaining to direct versus left treatments, direct versus right treatments and left versus right treatments respectively. N_1, N_2 and N_3 are v × b incidence matrices pertaining to direct treatments versus blocks, left treatments versus blocks and right treatments versus blocks respectively. Further $\mathbf{R}_\tau = \mathrm{diag}(r_1,r_2,\ldots,r_v)$; $\mathbf{R}_\delta = \mathrm{diag}(r_{11},r_{12},\ldots,r_{1v})$; $\mathbf{R}_\rho = \mathrm{diag}(r_{21},r_{22},\ldots,r_{2v})$, r_l (l =1, 2,…,v) being the number of times the l^{th} treatment appears in the design, r_{1l} (r_{2l}) being the number of times the treatments in the design has l^{th} treatment as left (right) neighbour. The 3v × 3v matrix **C** is symmetric, non-negative definite with zero row and column sums.

The information matrix for estimating the direct effects of treatments obtained from (1) is as follows:

$$\mathbf{C}_\tau = \mathbf{C}_{11} - \mathbf{C}_{12}\mathbf{C}_{22}^{-}\mathbf{C}_{21}, \quad (2)$$

where, $\mathbf{C}_{11} = \mathbf{R}_\tau - \frac{1}{k}\mathbf{N}_1\mathbf{N}_1'$ $\qquad \mathbf{C}_{12} = \begin{bmatrix} \mathbf{M}_1 - \frac{1}{k}\mathbf{N}_1\mathbf{N}_2' & \mathbf{M}_2 - \frac{1}{k}\mathbf{N}_2\mathbf{N}_3' \end{bmatrix}$

$$C_{22} = \begin{bmatrix} R_\delta - \frac{1}{k}N_2N_2' & M_3 - \frac{1}{k}N_2N_3' \\ M_3' - \frac{1}{k}N_3N_2' & R_\rho - \frac{1}{k}N_3N_3' \end{bmatrix}.$$

Similarly, the information matrices for estimating the right neighbour (C_ρ) and left neighbour (C_δ) effects can also be obtained. The adjusted sum of squares pertaining to each of the effect can then be worked out.

BLOCK DESIGNS BALANCED FOR NEIGHBOUR EFFECTS

Mead (1967) has given a mathematical model to estimate inter-plant competition based on correlation between neighbouring pairs of plants. Freeman (1967, 1969) used cyclic balanced incomplete block designs for directional and non-directional seed orchards. Gomez (1972) undertook a study to investigate the extent of varietal competition and its effects on comparison between varieties in transplanted lowland rice. Azais *et al.*, (1993) have presented 3 types of designs (Type 0 design, Type 1 design, Type 2 design) according to the number of treatments with v blocks of size v-1. Type 3 designs with v-1 blocks each of size v have also been obtained.

Definition 1: A block design with neighbour effects is **neighbour balanced** if every treatment has every other treatment appearing μ_1 times as a right and as a left neighbour.

Type 0 Designs

When v is a prime power but not a prime (4. 8, 9, 16), the construction is based on Galois fields. Since the method is not available, only the designs have been presented.

Example 1: For v = 4, following is the design:

3	1	2	3	1
2	0	3	2	0
1	3	0	1	3
0	2	1	0	2

-- ----, -- ...entioned that even though the designs are neighbour balanced, but these are not variance balanced for estimating the contrast pertaining to direct and neighbour effects.

Type 1 Designs

When v is prime or when v is odd and not a prime power (v = 3, 5, 7, 11, 13, 15), the design is constructed as follows:

For v = 2m + 1, consider the initial block of 2m numbers as follows:

0, 1, -1, 2, -2, ..., - (m-1), m (modulo 2m).

Developing this initial block cyclically results in neighbour balanced design.

Example 2: For m = 2, v = 5 (0, 1, 2, 3, 4). The initial block is **0 1 3 2** and the final design is as given:

2	**0**	**1**	**3**	**2**	0
3	1	2	4	3	1
4	2	3	0	4	2
0	3	4	1	0	3
1	4	0	2	1	4

Type 2 Designs

When v is even but not a power of 2 (v = 6, 10, 12, 14). It is modification of Type 1 design using an infinity element. A new element ∞ is inserted between any pair i, i′ of adjacent elements of Type 1 initial block, so long as i′-i is coprime to v-1 and each of the v-2 following blocks is constructed by adding 1 modulo v-1 to the preceeding block, with the rule "∞ + 1 = ∞". The last block is formed by taking the first plot of the other v-1 blocks.

Example 3: If v = 6, the initial block is **0 ∞ 1 3 2** mod 5; Last block is **0 1 2 3 4** and the final design is,

2	**0**	**∞**	**1**	**3**	**2**	0
3	1	∞	2	4	3	1
4	2	∞	3	0	4	2
0	3	∞	4	1	0	3
1	4	∞	0	2	1	4
2	0	1	2	3	4	0

Type 3 Designs

When v is prime (v = 3, 5, 7, 11, 13), the j^{th} block (j=1,2,..., v-1) of the neighbour balanced complete block design is

$0, j, 2j, \ldots, (v-1)j$ modulo v

Example 4: For v = 5, the final design is

4	0	1	2	3	4	0
3	0	2	4	1	3	0
2	0	3	1	4	2	0
1	0	4	3	2	1	0

The different incidence matrices in this case are of the following form:

$$\mathbf{M}_1 = \mathbf{J} - \mathbf{I} = \mathbf{M}_2 = \mathbf{M}_3 \ , \ \mathbf{N}_1 = \mathbf{J} = \mathbf{N}_2 = \mathbf{N}_3$$

The joint information matrix for estimating the direct, left and right effects of the treatments is

$$\mathbf{C} = (v-1)\left[\mathbf{I} - \frac{1}{v}\mathbf{J}\right] \otimes \mathbf{I}_3 + \left[\frac{1}{v}\mathbf{J} - \mathbf{I}\right] \otimes (\mathbf{J}_3 - \mathbf{I}_3) \ ,$$

$\otimes$ denotes the Kronecker product. The information matrix for estimating the direct effects of treatments after eliminating the effects of left and right neighbours is

$$\mathbf{C}_\tau = \frac{v(v-3)}{v-2}\left(\mathbf{I} - \frac{1}{v}\mathbf{J}\right)$$

It is seen that the direct effects of treatments are all estimated with the same variance and hence the design is variance balanced. Similarly the neighbour effects are also estimated with the same variance which is same as for direct effects.

Azais *et al.* (1993) have developed a programme for generation and randomization of neighbour balanced block designs discussed above. Druilhet (1999) studied optimality of circular neighbour balanced block designs of Azais *et al.* (1993) and showed that these designs are universally optimal for estimation of direct and neighbour effects of treatments.

Type 4 Designs

Tomar *et al.* (2005) have obtained general method of constructing incomplete neighbour balanced block designs. Let the number of treatments $v = sm + 1$ (prime or prime power) where $m > 3$. The sv blocks of size $k = m$ with replication of each treatment being sm, is obtained by developing the following initial blocks modulo v:

$x^{\alpha}, x^{\alpha+s}, x^{\alpha+2s}, \ldots x^{\alpha+(m-1)s}$

$\alpha = 0, 1, 2, \ldots, s-1$, where x is the primitive element of GF(sm+1).

This series of incomplete block design obtained is balance for the estimation of direct, left and right neighbour effects. The structure of the incidence matrices is as follows:

$$\mathbf{M}_1 = \mathbf{M}_2 = \mathbf{M}_3 = \mathbf{J} - \mathbf{I}; \quad \mathbf{N}_1\mathbf{N}_1' = \mathbf{N}_2\mathbf{N}_2' = \mathbf{N}_3\mathbf{N}_3' = (v-m)\mathbf{I} + (m-1)\mathbf{J}$$

$$\mathbf{C} = \begin{bmatrix} \frac{v(k-1)}{k}[\mathbf{I} - \frac{\mathbf{J}}{v}] & \frac{-v}{k}[\mathbf{I} - \frac{\mathbf{J}}{v}] & \frac{-v}{k}[\mathbf{I} - \frac{\mathbf{J}}{v}] \\ \frac{-v}{k}[\mathbf{I} - \frac{\mathbf{J}}{v}] & \frac{v(k-1)}{k}[\mathbf{I} - \frac{\mathbf{J}}{v}] & \frac{-v}{k}[\mathbf{I} - \frac{\mathbf{J}}{v}] \\ \frac{-v}{k}[\mathbf{I} - \frac{\mathbf{J}}{v}] & \frac{-v}{k}[\mathbf{I} - \frac{\mathbf{J}}{v}] & \frac{v(k-1)}{k}[\mathbf{I} - \frac{\mathbf{J}}{v}] \end{bmatrix}$$

Therefore the individual information matrices for estimating the direct effects ($\mathbf{C}_\tau$), left-neighbour effects ($\mathbf{C}_\delta$) and right-neighbour effects ($\mathbf{C}_\rho$) of treatments have been obtained as given below.

$$\mathbf{C}_\tau = \mathbf{C}_\rho = \mathbf{C}_\delta = \frac{v(k-3)}{k-2}\left(\mathbf{I} - \frac{\mathbf{J}}{v}\right)$$

Example 5: Let s = 2 and m = 5; therefore v = sm + 1 = 11. Developing the two initial blocks mod 11 by taking α = 0 and 1 along with the border plots, the following block design in 22 blocks of size 5 each is obtained:

3	1	4	5	9	3	1	6	2	8	10	7	6	2
4	2	5	6	10	4	2	7	3	9	0	8	7	3
5	3	6	7	0	5	3	8	4	10	1	9	8	4
6	4	7	8	1	6	4	9	5	0	2	10	9	5

7	5	8	9	2	7	5	10	6	1	3	0	10	6
8	6	9	10	3	8	6	0	7	2	4	1	0	7
9	7	10	0	4	9	7	1	8	3	5	2	1	8
10	8	0	1	5	10	8	2	9	4	6	3	2	9
0	9	1	2	6	0	9	3	10	5	7	4	3	10
1	10	2	3	7	1	10	4	0	6	8	5	4	0
2	0	3	4	8	2	0	5	1	7	9	6	5	1

Remark 1: In Type 4 design, if $s = 1$ and $m = v - 1$, the method reduces to obtaining Type 0 and Type 1 designs of Azais *et al.*, (1993) for v treatments in v blocks of size v-1 each. The information matrices for estimating the direct effects, left-neighbour effects and right-neighbour effects of treatments with $k = v-1$ reduces to

$$\mathbf{C}_\tau = \mathbf{C}_\rho = \mathbf{C}_\delta = \frac{v(v-4)}{v-3}\left(\mathbf{I} - \frac{\mathbf{J}}{v}\right).$$

Remark 2: All the designs obtained above are **minimally balanced** in the sense that every treatment has every other treatment as neighbour on both sides once, i.e. $\mu_1=1$.

Remark 3: Type 1, Type 3 and Type 4 designs have the property that, for each ordered pair of distinct treatments, there is exactly one plot that has the first chosen treatment as left neighbour and second chosen treatment as right neighbour. Such designs are said to be **neighbour balanced at distance 2**. An advantage of these designs is that comparisons between two treatments are all estimated with the same precision.

Remark 4: Type 0, Type 1, Type 2 and Type 4 designs are indeed BIB designs if border plots are not considered.

RANDOMIZATION OF CIRCULAR NEIGHBOUR BALANCED DESIGNS

The method of randomization was introduced by Azais (1987). It consists of performing the following four operations:

i. Total inter-block randomization

ii. Independently within each block, random circular permutation of plot

iii. Random allocation of one element of (0, 1, ...,v-1) to each treatment

iv. Addition of border plots so that the circularity condition is met

Example 6: Consider the Type 1 design for v = 5. In step (i), the original blocks move to positions (4, 2, 3, 5, 1) respectively. In step (ii), the plots within the different blocks in that new order are permutated circularly. According to step (iii), treatments are associated with the elements 4, 2, 3, 0, 1 respectively with original treatments 0, 1, 2, 3, 4 respectively. Finally border plots are added to meet the circularity.

0	1	3	2		4	0	2	1		4	0	2	1
1	2	4	3		1	2	4	3		3	1	2	4
2	3	0	4		2	3	0	4		3	0	4	2
3	4	1	0		0	1	3	2		2	0	1	3
4	0	2	1		3	4	1	0		1	0	3	4
Initial design					After step (i)					After step (ii)			

1	4	3	2		1	2	4	0	1	2
0	2	3	1		2	3	1	0	2	3
0	4	1	3		0	3	4	2	0	3
3	4	2	0		3	0	4	1	3	0
2	4	0	1		2	1	4	3	2	1
After step (iii)					Final design					

BLOCK DESIGNS PARTIALLY BALANCED FOR NEIGHBOUR EFFECTS

In order to achieve the property of balance, the size of the design, in terms of the number of units required, becomes large. In many experiments, it may not be possible to have as many replications as is needed to achieve balancedness. Therefore, block designs partially balanced for neighbours are required.

Definition 2: A block design with neighbour effects is said to be **partially neighbour balanced** based on m-class association scheme if two treatments θ and ϕ that are mutually u^{th} associates ($u = 1, 2, \ldots, m$) appear as neighbours (left and right) μ_{1u} times.

Jaggi *et al.* (2006) have obtained series of block designs partially balanced for neighbour effects.

Method 1: When $v = 2^p$

Arrange $v = 2^p$ treatments on the circumference of a circle. The contents of complete blocks of the design for studying competition effects among neighbouring units are obtained by writing in systematic order treatments at intervals of $1,3,5,\ldots,2^p-1$. The first block is formed by taking all the treatments at intervals of one in the circle; the second block is obtained by taking all treatments at interval of three in the circle; and so on, the $(2^{p-1})^{th}$ block is obtained by taking all treatments at interval of 2^p-1 in the circle. Then by including border plots as given above, a block design with parameters $v = 2^p = k$, $r = b = 2^{p-1}$ is obtained which is partially balanced for neighbouring competition effects with two associate classes.

The association scheme for direct effects of treatments in relation to the neighbours is as follows:

The l^{th} treatment ($l = 1,2,\ldots,v = 2^p$) has treatments $l+1$, $l+3$, $l+5,\ldots,l+2^p-1$ (mod v) as first associates and remaining as second associates. In other words two treatments are first associates if they appear as neighbours, both left and right, and second associates otherwise. Here, the number of first associates = $\frac{v}{2}$ and the number of second associates = $\frac{v}{2}-1$. Further $\mu_{11} = 1$ and $\mu_{12} = 0$.

Example 7: Let p = 3 i.e. v = 8. The following four blocks of the design are obtained by taking a difference of 1, 3, 5 and 7. The final design is a block design partially balanced with two associate classes.

7	0	1	2	3	4	5	6	7	0
5	0	3	6	1	4	7	2	5	0
3	0	5	2	7	4	1	6	3	0
1	0	7	6	5	4	3	2	1	0

Treatment 0 has treatments 1, 3, 5 and 7 as first associates as these are appearing as neighbours and 2, 4, 6 as second associates. Likewise, for other treatments also the associates pertaining to neighbour can be seen.

If the interest is in studying the neighbour effects, then the above method would give rise to block design for neighbour effects partially balanced for other effects with 3-associate classes. For a treatment l ($l=1,2,\ldots,2^p$), the first associates are l+1, l+3, l+5,..., $l+2^p-1$; the second associates are l+2, l+6, l+10,..., $l+2^p-2$; and the third associates are l+4, l+8,..., $l+2^p-4$ (mod v).

Method 2: When v is prime

Arrange the v treatments on the circumference of a circle. The contents of $(v-1)/2$ complete blocks of the design for studying competition effects among neighbouring units are obtained by writing in systematic order treatments at intervals of 1,2,..., $(v-1)/2$. The first block is formed by taking all the treatments at intervals of one in the circle; the second block is obtained by taking all the treatments at interval of two in the circle; and so on, the $(v-1)/2)^{th}$ block is obtained by taking all treatments at interval of $(v-1)/2$ in the circle. The design so obtained is a partially balanced design with $(v-1)/2$ associate classes for left and right neighbour effects with parameters v, b = r = $(v-1)/2$, k = v. The joint information matrix for estimating the direct, left and right effects of v treatments in $(v-1)/2$ blocks of size v each is as obtained below:

$$\mathbf{C} = \begin{bmatrix} \left(\frac{v-1}{2}\right)\left[\mathbf{I} - \frac{1}{v}\mathbf{J}\right] & \mathbf{M}_1 - \frac{v-1}{2v}\mathbf{J} & \mathbf{M}_2 - \frac{v-1}{2v}\mathbf{J} \\ \mathbf{M}_1' - \frac{v-1}{2v}\mathbf{J} & \left(\frac{v-1}{2}\right)\left[\mathbf{I} - \frac{1}{v}\mathbf{J}\right] & \mathbf{M}_3 - \frac{v-1}{2v}\mathbf{J} \\ \mathbf{M}_2' - \frac{v-1}{2v}\mathbf{J} & \mathbf{M}_3' - \frac{v-1}{2v}\mathbf{J} & \left(\frac{v-1}{2}\right)\left[\mathbf{I} - \frac{1}{v}\mathbf{J}\right] \end{bmatrix} \quad (3)$$

Any treatment l (l=1,2,...,v) is said to be u^{th} $\{u=1,2,\ldots, (\nu-1)/2\}$ associate of l' if $l'=l \pm u$, mod(v). As a result the number of u^{th} associates of any treatment is 2. In the given situation, the associates are with respect to the neighbours. The number of associate classes would be $(\nu-1)/2$. The number of times the u^{th} associates of a treatment l appears as $(l-u)^{th}$ left neighbour and $(l+u)^{th}$ right neighbour in the design is always one.

Example 8: For v = 7, the complete block design with border plots in 3 blocks partially balanced for neighbours with 3-associate classes is as given below:

6	0	1	2	3	4	5	6	0
5	0	2	4	6	1	3	5	0
4	0	3	6	2	5	1	4	0

Particular case

When b = v -1, then the design obtained is a Type 3 design for left and right neighbour effects with parameters v, b = v-1, r = v-1, k = v as given by Azais *et al.* (1993).

Method 3:

This method gives rise to a series of incomplete block designs partially balanced for neighbouring effects is described. Let the number of treatments v = 2km+1 (prime or prime power) where k >3. The mv blocks of size k with replication of each treatment being km, is obtained by developing the following initial blocks modulo v and including the border units:

$x^{2\alpha}\ x^{2(m+\alpha)}, x^{2(2m+\alpha)}, \ldots, x^{2(mk-m+\alpha)}; \quad \alpha = 0, 1, 2, \ldots, m-1,$

where x is the primitive element of GF(2km+1). The incomplete block design obtained is partially neghbour balanced with two associate classes for the estimation of direct, left and right neighbour effects. First associates of treatments are those that appear as neighbours (left and right) once and second associates do not occur as neighbours. Here, the number of first associates = number of second associates = $(v-1)/2$. Further, $\mu_{11} = 1$ and $\mu_{12} = 0$.

Example 9: Let m = 2, k = 4, therefore v = 2mk+1 = 17. Developing the two initial blocks mod 17 by taking α = 0 and 1, a partially neighbour balanced block design in 34 blocks with replication of each treatment being 8, for estimating direct effects as well as for estimating neighbour effects is obtained as below:

4	1	13	16	4	1	2	9	15	8	2	9
5	2	14	0	5	2	3	10	16	9	3	10
6	3	15	1	6	3	4	11	0	10	4	11
7	4	16	2	7	4	5	12	1	11	5	12
8	5	0	3	8	5	6	13	2	12	6	13
9	6	1	4	9	6	7	14	3	13	7	14
10	7	2	5	10	7	8	15	4	14	8	15
11	8	3	6	11	8	9	16	5	15	9	16
12	9	4	7	12	9	10	0	6	16	10	0
13	10	5	8	13	10	11	1	7	0	11	1
14	11	6	9	14	11	12	2	8	1	12	2
15	12	7	10	15	12	13	3	9	2	13	3
16	13	8	11	16	13	14	4	10	3	14	4

0	14	9	12	0	14	15	5	11	4	15	5
1	15	10	13	1	15	16	6	12	5	16	6
2	16	11	14	2	16	0	7	13	6	0	7
3	0	12	15	3	0	1	8	14	7	1	8

Here treatment 1 has 4, 6, 7, 8, 11, 12, 13, 15 as first associates as they appear as neighbour, both on left and right side and others are second associates. Likewise, for other treatments also the associates can be obtained.

Method 4: Azais *et al.* (1993) have given a series of partially neighbour balanced block designs which is slight modification of the method given by Williams (1949). If v is odd then let n = (v-1)/2. The first block is,

0 (2n-1) 1 (2n-2) 2 (2n-3) ... n ∞

Develop the initial block cyclically modulo 2n.

Example 10: Let v = 9 (0, 1, 2, 3, 4, 5, 6, 7, ∞), then n = 4. Developing initial block using above series, following design is obtained:

∞	0	7	1	6	2	5	3	4	∞	0
∞	1	0	2	7	3	6	4	5	∞	1
∞	2	1	3	0	4	7	5	6	∞	2
∞	3	2	4	1	5	0	6	7	∞	3

If v is even, let n = (v-2) / 2. The first block is,

$$\underbrace{0\ (2n-1)\ 1\ (2n-2)\ 2\ (2n-3)\ \ldots}_{n \text{ terms}}\ *\ \ldots\ \underbrace{(n-1)\ n\ \infty}_{n \text{ terms}}$$

Example 11: Let v = 10 (0, 1, 2, 3, 4, 5, 6, 7, *, ∞), then n = 4. Developing initial block as given above, following design is obtained:

∞	0	7	1	6	*	2	5	3	4	∞	0
∞	1	0	2	7	*	3	6	4	5	∞	1
∞	2	1	3	0	*	4	7	5	6	∞	2
∞	3	2	4	1	*	5	0	6	7	∞	3

Jaggi *et al.* (2007) studied optimal complete block designs for neighbouring competition effects. Tomar (2007) studied some aspects of neighbour balanced block designs for correlated observations. Pateria *et al.* (2007) proposed a series of incomplete non-circular block designs for competition effects.

Sarika and Sharma (2008a; 2008b) studied response surface designs incorporating neighbour effects. Sarika and Sharma (2008a) studied first order response surface model with neighbour effects from immediate left and right neighbouring units and the conditions have been derived for the orthogonal estimation of coefficients of this model. The variance of estimated response has also been obtained and conditions for first order response surface model with neighbour effects to be rotatable have been obtained. A method of obtaining designs satisfying the derived conditions was given.

Second order response surface model in which the experimental units experience the neighbour effects has also been studied by Sarika and Sharma (2008b). Conditions have been derived for the estimation of coefficients of second order response surface model and a method of constructing designs for fitting second order response surface in the presence of neighbour effects has been developed. The designs so obtained are found to be rotatable.

CONCLUSIONS

In this paper, some series of neighbour balanced block designs have been described. The incomplete neighbour balanced block designs discussed are in fact BIB designs. The additional border units have been added for neighbour balance. These designs help in estimating the direct and neighbour (left and right) effects of treatments under the situation when the response on a given plot is affected by treatments on neighbouring plots as well as by the treatment applied to that plot. The series of partially neighbour balanced incomplete block designs discussed is simple PBIB design with 2-associate classes following cyclic association scheme if border units are not considered. The partially neighbour balanced block designs require less number of units without much losing in efficiency of treatment effects.

REFERENCES

Azais, J.M., R.A. Bailey and H. Monod (1993). A catalogue of efficient neighbour designs with border plots. *Biometrics*, 49: 1252-1261.

Bailey, R.A and P. Druilhet (2004). Optimality of neighbour balanced designs for total effects. *The Annals of Statistics*, 32: 1650-1661.

Druilhet, P. (1999). Optimality of neighbour balanced designs. *Journal of the Statistical Planning and Inference*, 81: 141–152.

Freeman G.H. (1969). The use of cyclic balanced incomplete block designs for non-directional seed orchards. *Biometrics*, 23: 561-571.

Freeman G.H. (1979). Some two-dimensional designs balanced for nearest neighbours. *Journal of the Royal Statistical Society*, 41(B): 88-95.

Gomez, K.A. (1972). Border effects of rice experimental plots II. Varietal competition. *Journal of Exp. Agric.*, 8: 295-298.

Jaggi S., V.K. Gupta and J. Ashraf (2006). On block designs partially balanced for neighbouring competition effects. *Journal of the Indian Statistical Association*, 44(1): 27-41.

Jaggi S., C. Varghese and V.K. Gupta (2007). Optimal block designs for neighbouring competition effects. *Journal of Applied Statistics*, 34(5): 577-584.

Mead, R. (1967). A mathematical model for the estimation of inter-plant competition. *Biometrics*, 23: 189-205.

Pateria, D.K., S. Jaggi, C. Varghese and M.N. Das. (2007). Incomplete non-circular block designs for competition effects. *Statistics and Applications*, 5(1&2): 5-14.

Sarika, J.S. and V.K. Sharma (2008a). First order rotatable designs incorporating neighbour effects. *ARS Combinatoria*. In press.

Sarika, J.S. and V.K. Sharma (2008b). Second order response surface model with neighbour effects. *Commn. in Statist.: Theory and Methods*. In press.

Tomar, J.S., J. Seema and C. Varghese (2005). On totally balanced block designs for competition effects. *Journal of Applied Statistics*, 32(1): 87-97.

Tomar, J.S. (2007). Design and analysis of agricultural experiments under interference and dependent Observations. *Ph.D. Thesis*, IARI, New Delhi.

Williams, E.J. (1949). Experimental designs balanced for the estimation of residual effects of treatments. *Australian Journal of Scientific Research*, A2:1 149-168.