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Time to Event Analysis of Multiple Failure Modes: An Application to Mobile Phone Failures

K. GALAPPATHTHI

Department of Statistics, University of Colombo, Sri Lanka

ABSTRACT

Consider a device which could fail due to one or more reasons. The reliability of the device depends on the occurrence of these failure modes. A majority of the statistical approaches for analyzing multiple failure modes have been developed for the medical field. However, little work has been done on developing statistical methods for analyzing multiple failure data in the field of reliability engineering. The main objective of this study is to examine and further develop the available statistical methods for analyzing multiple failure data for the field of reliability engineering by way of an application which considers the reliability of mobile phones under three failure modes. Additional objectives are to classify five available mobile phone brands and identify best phone brand. Another important objective is the assessment of the worthiness of the standard warranty period. In the example, a mobile phone is considered as a 'series system' with three components which are the three failure modes. For one particular brand, the three failure modes are modeled using the Weibull distribution and when considering one failure type the other two failure modes are taken as censored. The overall system reliabilities for each brand are computed to compare the five mobile phone brands. Brand 3 is identified as the best phone brand. The methodology used in analyzing multiple modes of failure in this research study would enhance the field of reliability data analysis. The findings of this study provide insight into reliability of different mobile phone brands and certain design characteristics that eliminate or minimize occurrence of failures. However none of these mobile phone brands are very reliable and thus a warranty period of one year is not worthwhile for the sales company. These results can be used by mobile phone manufacturers to improve the reliability of the mobile phones.

Keywords: Multiple failure modes, Reliability, Series system, Weibull distribution, Warranty period

INTRODUCTION

It is a known fact that any device, equipment or system faces the event of failure due to one or more reasons. These reasons are referred to as failure modes or failure types. It is obvious that more components/parts in any equipment or device would increase the risk of occurrence of at least one particular failure. A mobile phone which is made-up of several components like keypad, software, battery, vibration motor, connection wires and so on is a good example of a multi-component system involving multiple modes of failure. The identification of the individual failure modes and the evaluation of the respective probabilities are problems of this sort of systems. The manufacturers of these devices are in a constant race to ensure or rather improve the reliability of their products to compete on a global scale with increasing varied customer expectations. Therefore becoming aware of possible failure modes is vital as manufacturers are liable for the reliability of their devices. This is where the failures and different failure modes in the context of reliability data analysis is important. Failure analysis is an important function to all engineering disciplines. In any case, one must determine the cause and type of failure to prevent future occurrences, and/or to improve the performance of the device. Likewise in the process of designing new models of these devices, the past failure types and failures need to be taken into account to upgrade its performance and hence the reliability. Therefore studying about different failure modes helps early identification and elimination of potential failure types thus reducing the potential for warranty concerns. These studies also help to identify remedial actions that reduce cumulative impacts of life cycle consequences/risks from a failure.

The main objective of this study is to examine and further develop the available statistical methods for analyzing multiple failure data for the field of reliability engineering by way of an application which considers the reliability of mobile phones under three failure modes. This involves the modelling of different types/modes of mobile phone failures. The different mobile phone brands will be classified according to their failure rates. Identification of certain design characteristics which contribute to failures so as to minimize or eliminate those effects is an additional objective of the study. The industry standard of warranty for mobile phones is one year. Thus assessing the worthiness of a one year warranty period is another important objective. The industry is also keen to identify the 5^{th} percentile which is the time at which 5% of the phones fail. Finally the best phone brand among the brands involved in this research study will be identified.

The data consists of multiple failure modes of five different mobile phone brands. The names of the brands will not be revealed in accordance with confidentiality of data providers.

Section 2 consists of materials and methods used/developed in this study. Section 3 comprises of an example where the methodology explained in section 2 is applied to a data set consisting of multiple failure types of mobile phones. This section includes results given by the data and conclusions drawn. Section 4 presents a discussion of the entire study and this involves the important conclusions drawn from the results. This is followed by a list of references.

MATERIALS AND METHODS

Review of Some Methodology Available in the Literature

Initially it was of interest to find out whether the multiple failure data could be modeled by a familiar location-scale or log-location-scale distribution and if so to determine this distribution. Three commonly used and practically plausible distributions, namely, normal, lognormal and weibull distributions were examined. The quantities of interest in reliability analysis with respect to this study are 'probability of failure', 'survival probability' and 'p^{th} quantile'. Meeker and Escobar (1998) describe the properties of the three considered distributions in detail. Probability plots (Ebeling, 1997; Meeker and Escobar, 1998; O'Connor, 2002) were used to check the adequacy of each distribution considered and to select the most appropriate distribution from the three distributions considered. This technique requires the cumulative distribution function to be linearized in such a way that the plots of the transformed variables are linear. Two techniques were used to decide on the goodness of fit of the selected distribution. The first is the probability plot technique. Departures from a straight line suggest a lack of fit of the fitted distribution. Additionally simultaneous confidence bands (Meeker and Escobar, 1998) should be given on the probability plot so as to measure the sampling uncertainty simultaneously over a range of values of time so as to quantify the magnitude of observed departures from the fitted parametric model. Lack of fit is strongly indicated if the departures from the straight line are beyond these confidence bands. The second technique used is the Mann's goodness of fit test for the weibull distribution (Ebeling, 1997).

Ebeling (1997), Meeker and Escobar (1998) and O'Connor (2002) describe how failure data can be modeled using the weibull distribution. The linearized weibull model is

$$log(t_p)= \mu+loge[-loge(1-p)]^{\sigma} \qquad (1)$$

In the presence of a single factor with J levels equation (1) becomes

$log(t_p) = \beta_0 + \beta_k + loge[-loge(1-p)]^{\sigma}$; $\quad k=1,2,......J$

Where β_0 is the intercept and can be interpreted as the value of μ for the null model, β_k is the effect of the k^{th} level of the factor k=1,2,......J and σ is the scale parameter.

The method of maximum likelihood (ML) is used for estimating the model parameters.

The likelihood function of the weibull distribution is,

$$L(\mu,\sigma) = \prod_{i=1}^{n} \left\{ \frac{1}{\sigma t_i} \phi_{sev} \left[\frac{\log(t_i) - \mu}{\sigma} \right] \right\}^{\delta_i} \times \left\{ 1 - \Phi_{sev} \left[\frac{\log(t_i) - \mu}{\sigma} \right] \right\}^{1-\delta_i} \quad (2)$$

Where $\delta_i=1$ if t_i is an exact observation and 0 if t_i is right censored. Here ϕ_{sev} and Φsev are standardized probability density function and cumulative distribution function of the smallest extreme value distribution respectively.

Suppose model 1 is the null Weibull model, model 2 is the Weibull model with a single factor with J levels and model 3 refers to J separate Weibull models for each level of the factor. Let L (Q) be the maximized log likelihood under model Q. To test H_0 *versus* H_1 where $H_0 \subset H_1$ the likelihood ratio test statistic (McCullah and Nelder, 1989), $-2\times[L(H_1) - L(H_0)]$, which has a Chi-Square distribution under the null hypothesis H_0 with $(\dim(H_1) - \dim(H_0))$ degrees of freedom is used. The strategy used in this paper for model selection is to first compare Model 2 with Model 1 in order to determine whether the factor has an important significance on failure time. If it has no significance Model 1 is selected else Model 2 is selected. Then the next step is to identify whether the scale parameter for each level can be considered as a constant (Model 2) or whether it varies with the level (Model 3). In order to do this Model 2 is compared with Model 3. If the corresponding likelihood ratio test statistic is not significant Model 2 can be selected in preference to Model 3 and constant scale parameter between levels concluded. Otherwise Model 3 has to be selected and a separate model (different scale parameters) for each level of the factor has to be assumed.

Meeker and Escobar (1998) and Cox and Hinkley (1974), describe the calculation of Cox-Snell residuals. For testing the weibull assumption standardized Cox-Snell residuals are plotted on a Weibull probability plot. Here $\log[-\log(1-p)]$ *versus* the ordered Cox-Snell residuals will plot as a straight line if the distribution is Weibull. If the plot is linear through the origin with a slope=1, the fitted model is adequate. Any departures from this indicate lack of fit.

Development of Methodology by Authors for Determining Characteristics for a Series System Under the Weibull Distribution

In this section the authors develop methods for determining failure probabilities, percentiles, confidence limits and mean time to failure (MTTF) for a series system (Bentley, 1993) assuming that for each level of the factor and for each failure mode the failure data follows a weibull distribution. It is also assumed that the different failure modes are independent of each other and if one type of failure occurs the entire system fails. These are reasonable assumptions to make for the set of data which is used in this study, however these should be checked if there are any doubts as to their validity.

Consider a series system (Bentley, 1993) with J failure modes each representing a failure category and one explanatory variable with I levels. The survival function for the j^{th} failure type of the i^{th} brand under the weibull distribution can be denoted by,

$$R_{ij}(t) = \exp\left[- \exp\left(- \mu_{ij} / \sigma_{ij}\right) t^{1/\sigma_{ij}} \right]; \; i = 1, 2,.., I \text{ and } j = 1, 2....J \quad (3)$$

The reliability of the system for the i^{th} brand assuming that each type of failure occurs independently, is,

$$\hat{R}_i(t) = \prod_j^J \hat{R}_{ij}(t) \quad (4)$$

Therefore the failure probability of the series system for the i^{th} brand is,

$$\hat{F}_i(t) = 1 - \hat{R}_i(t) = 1 - \prod_j^J \hat{R}_{ij}(t)$$

Let p_i be the failure probability that p percent of the devices for the i^{th} level of explanatory variable will fail after time t_p.

$$\text{i.e. } p_i = \hat{F}_i(t_p) = 1 - \hat{R}_i(t_p) = 1 - \prod_j^J \exp\left[-\exp(-\mu_{ij}/\sigma_{ij})\, t^{1/\sigma_{ij}}\right]$$

$$\log_e(1-p_i) = \sum_j^J \left[-\exp(-\mu_{ij}/\sigma_{ij}) t^{1/\sigma_{ij}}\right] \qquad (5)$$

The p^{th} percentile (t_p) for the i^{th} brand can be found by solving the equation (5) using Bisection method (Veerarajan and Ramachandran, 2004).

For the i^{th} brand the logit transformed $100(1-\alpha)$ % Confidence Interval for $F_i(t)$ (Meeker and Escobar, 1998) is,

$$\left[\frac{\hat{F}_i(t)}{\hat{F}_i(t)+\hat{R}_i(t)*W}, \frac{\hat{F}_i(t)}{\hat{F}_i(t)+\hat{R}_i(t)/W}\right] \qquad (6)$$

$$\text{where } W = \left[\frac{Z_{1-\alpha/2}\, S.e.(\hat{F}_i(t))}{\hat{F}_i(t)*\hat{R}_i(t)}\right]$$

In order to determine the confidence interval in equation (6) an estimate of the standard error of $\hat{F}_{ij}$ (t) is required. This is derived in the following section.

As $\hat{F}_i(t) = 1 - \hat{R}_i(t)$, $S.e.(\hat{F}_i(t)) = S.e.(1 - \hat{R}_i(t))$. Due to the device being a series system $\hat{R}_i(t) = \prod_j^J \hat{R}_{ij}(t)$ and thus $\log(\hat{R}_i(t)) = \sum_j^J \log(\hat{R}_{ij}(t))$. Since $\hat{R}_i(t)$'s are independent, $Var[\log(\hat{R}_i(t))] = \sum_j^J Var[\log(\hat{R}_{ij}(t))]$. For a function g of θ and γ the variance of $g(\theta, \gamma)$ can be obtained from the Delta method (Oehlert, 1992), and is expressed as

$$Var(\theta,\gamma) = \left[\frac{\partial g}{\partial \theta}\right]^2 Var(\theta) + \left[\frac{\partial g}{\partial \gamma}\right]^2 Var(\gamma) + 2\left[\frac{\partial g}{\partial \theta}\right]\left[\frac{\partial g}{\partial \gamma}\right]\text{cov}(\theta,\gamma) \qquad (7)$$

From result (7) therefore $Var[\log(\hat{R}_{ij}(t))] = \left[\frac{1}{\hat{R}_{ij}(t)}\right]^2 Var(\hat{R}_{ij}(t))$. Also Since $\hat{R}_{ij}(t)$ is a function of $\hat{\mu}_{ij}$ *and* $\hat{\sigma}_{ij}$ by using the result given in (7), result (8) is obtained.

$$Var[\hat{R}_{ij}(t)] = \left[\frac{\partial \hat{R}_{ij}}{\partial \mu_{ij}}\right]^2 Var(\hat{\mu}_{ij}) + \left[\frac{\partial \hat{R}_{ij}}{\partial \sigma_{ij}}\right]^2 Var(\hat{\sigma}_{ij}) + 2\left[\frac{\partial \hat{R}_{ij}}{\partial \mu_{ij}}\right]\left[\frac{\partial \hat{R}_{ij}}{\partial \sigma_{ij}}\right]\mathrm{cov}(\hat{\mu}_{ij}, \hat{\sigma}_{ij}) \tag{8}$$

By differentiating $\hat{R}_{ij}(t) = \exp[-\exp(-\hat{\mu}_{ij} / \hat{\sigma}_{ij})\, t^{1/\hat{\sigma}_{ij}}]$(equation 3) with respect to μ_{ij} *and* σ_{ij} respectively $\left[\frac{\partial \hat{R}_{ij}}{\partial \mu_{ij}}\right]$ and $\left[\frac{\partial \hat{R}_{ij}}{\partial \sigma_{ij}}\right]$ can be-found as follows.

$$\frac{\partial \hat{R}_{ij}}{\partial \mu_{ij}} = \hat{R}_{ij}(t) \log_e \hat{R}_{ij}(t)(-1/\hat{\sigma}_{ij}) \tag{9}$$

$$\frac{\partial \hat{R}_{ij}}{\partial \sigma_{ij}} = \frac{1}{\hat{\sigma}_{ij}^{\,2}} \hat{R}_{ij}(t) \log_e \hat{R}_{ij}(t)[\mu_{ij} - \log_e(t)] \tag{10}$$

The variances and co-variances of $\hat{\mu}_{ij}$ *and* $\hat{\sigma}_{ij}$ are given by the statistical packages when fitting separate weibull models for each failure type and level of the explanatory variable.

Once $Var[\log_e \hat{R}_{ij}(t)]$ is calculated, $Var[\hat{F}_{ij}(t)]$ can be found by

$$[Var[\hat{F}_{ij}(t)] = Var[\hat{R}_{ij}(t)] = [\hat{R}_{ij}(t)]^2 Var[\log_e \hat{R}_{ij}(t)] \tag{11}$$

The mean time to failure (MTTF) of the system is defined as $MTTF = \int_0^\infty R(t)$. Thus the MTTF for the j^{th} failure type of the i^{th} brand is,

$$MTTF_{ij} = \int_0^{\infty} \exp\left[- \exp\left(- \mu_{ij} / \sigma_{ij} \right) t^{1/\sigma_{ij}} \right] dt \qquad (12)$$

MTTF for the i^{th} level of the explanatory variable of the series system is,

$$MTTF_i = \int_0^{\infty} \prod_j^J \exp\left[- \exp\left(- \mu_{ij} / \sigma_{ij} \right) t^{1/\sigma_{ij}} \right] dt \qquad (13)$$

The integrals in (12) and (13) are evaluated using numerical integration (Veerarajan, Ramachandran, 2004).

Example

Description

In order to illustrate the methodology, the theory is applied to multiple failure records of mobile phones. This data was provided by a well-known automation company in Sri Lanka. Five different mobile phone brands (Brand 1, Brand 2, Brand 3, Brand 4, Brand5) which are available in Sri Lanka are considered and the names of the brands will not be revealed in this dissertation in accordance with confidentiality of data providers. A total of 976 failure records after removing repeat failures consist the dataset of this study as this is a time to event analysis. The variables 'Phone Brand', 'Time to Failure', 'Duration of Repair' and 'Failure Type' are measured for each failure record. The different failure types are categorized in to three groups (Type 1, Type 2, and Type 3) based on the severity of the original failure mode after consulting with the providers of data. The Type 1 group consists minor failures like keypad faults, speaker faults and so on. The Type 2 group is of moderate severity with failure types like poor signal reception and charging faults. The most sever failures like drop and liquid damaged units comprise the Type 3 category. These three failure type groups are considered to be relatively independent to each other and if at least one of these failures occurs, the phone is brought for repairs. The system which best describes the scenario of this study, is a 'Series System' (Bentley, 1993). The system diagram is given in figure1. The dataset used is given in table 1.

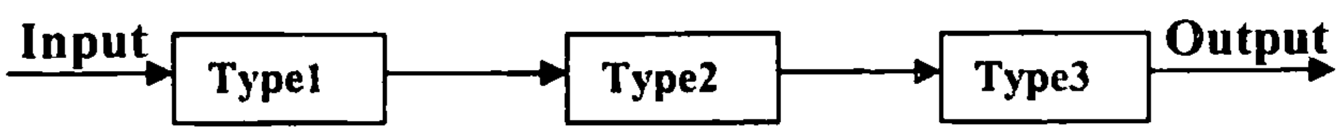


Figure 1: The system diagram

Table 1: Data used in study (Number of failures of each type and brand)

Failure Type \ Brand	Brand 1	Brand 2	Brand 3	Brand 4	Brand 5
Type 1	135	194	33	50	18
Type 2	93	125	16	36	19
Type 3	67	102	42	30	16

Descriptive and Univariate Analysis

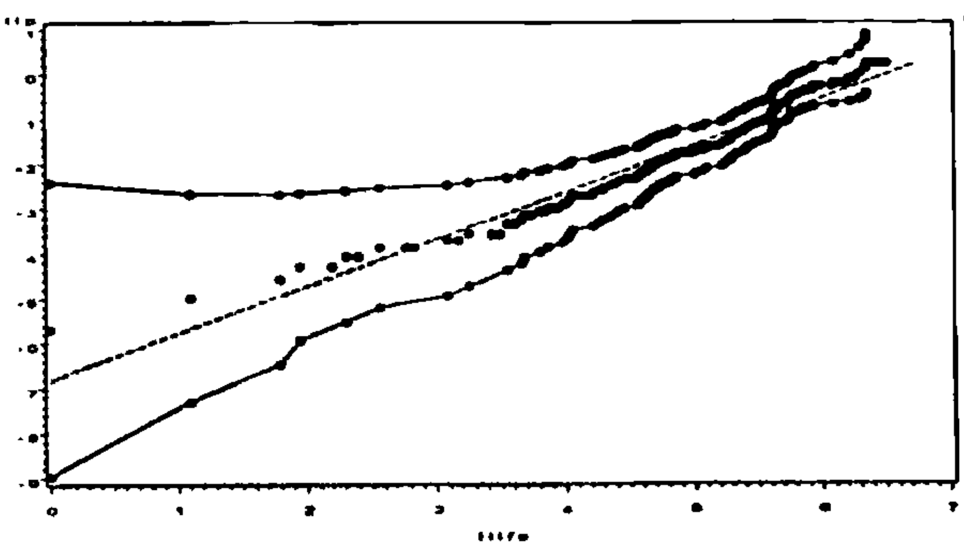
Table 2 gives the proportion of failure records for the entire data set and for each failure mode for the five brands of mobile phones.

Table 2: Distribution of brand-wise failure types

Failure Type \ Brand	Brand 1	Brand 2	Brand 3	Brand 4	Brand 5	Overall
Type 1	46%	46%	36%	43%	34%	44%
Type 2	31%	30%	18%	31%	36%	30%
Type 3	23%	24%	46%	26%	30%	26%

Overall the failure Type 1 has the highest occurrence compared to types 2 and 3. Type 3 failure has the smallest frequency of occurrence. For Brand 1, Brand 2 and Brand 4 the occurrence of Type 1 is higher compared to Type 2 and 3. Whereas for the Brand 3 occurrence of Type 3 failure is higher and for the Brand 5 frequency of Type 2 is higher. But overall when each failure type is considered, almost similar proportions of failures were recorded for each mobile phone brand.

The probability plots are used to identify the distribution of each failure type for each brand. In this study normal, lognormal and weibull probability plots were drawn and the linearity of the drawn plot was used to suggest that the data follows the assumed distribution. Compared to normal and lognormal probability plots the Weibull probability plots seemed to be the best as linearity was depicted in majority of weibull plots than in other two types of probability plots. In every Weibull plot data are within the simultaneous confidence bands. Figure 2 (2.a-2.o) gives the Weibull probability plots for each of 3 failure types for the five mobile phone brands. (2a-brand1, type1; 2b-brand1, type2; 2c-brand1, type3; 2d-brand2, type1 and so on). The linearity of these plots and the fact that the plots fall within the simultaneous confidence bands indicate the appropriateness of the Weibull distribution for modelling the data.



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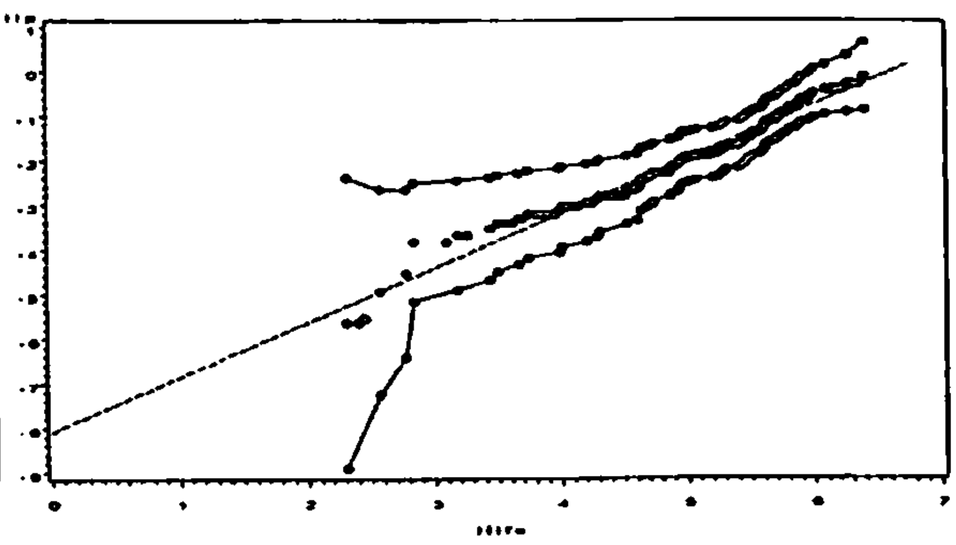

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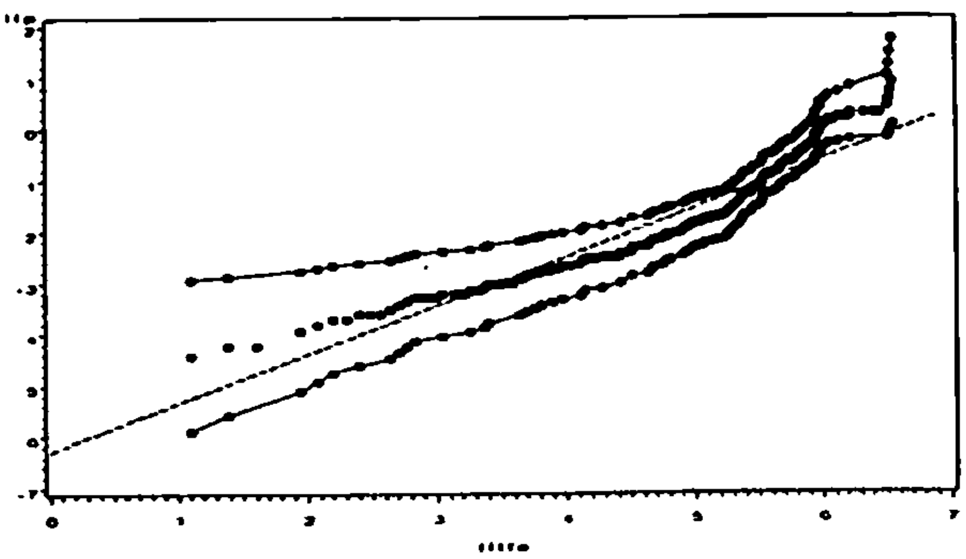

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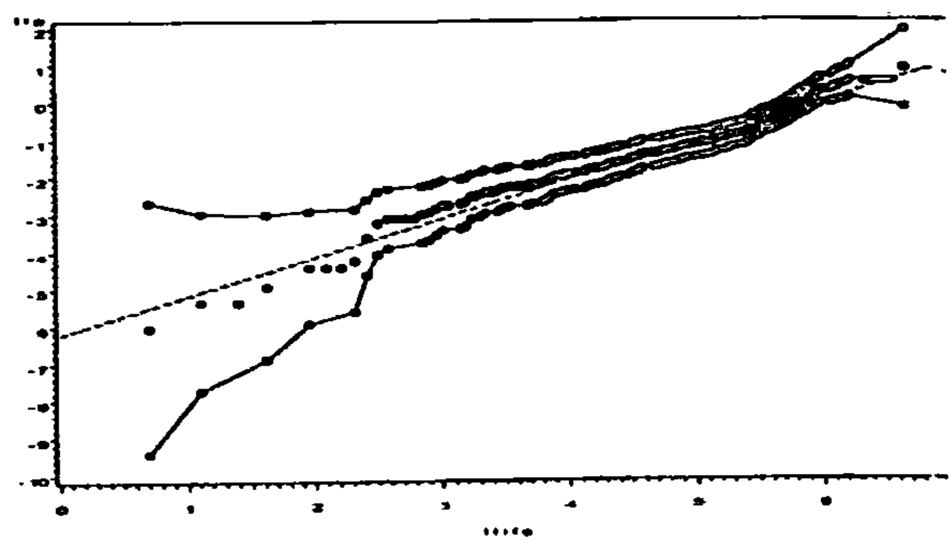

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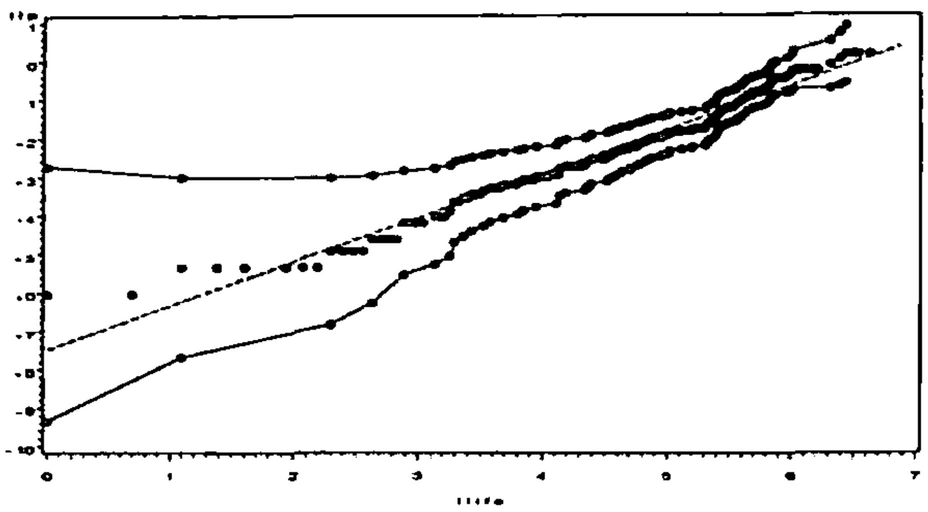

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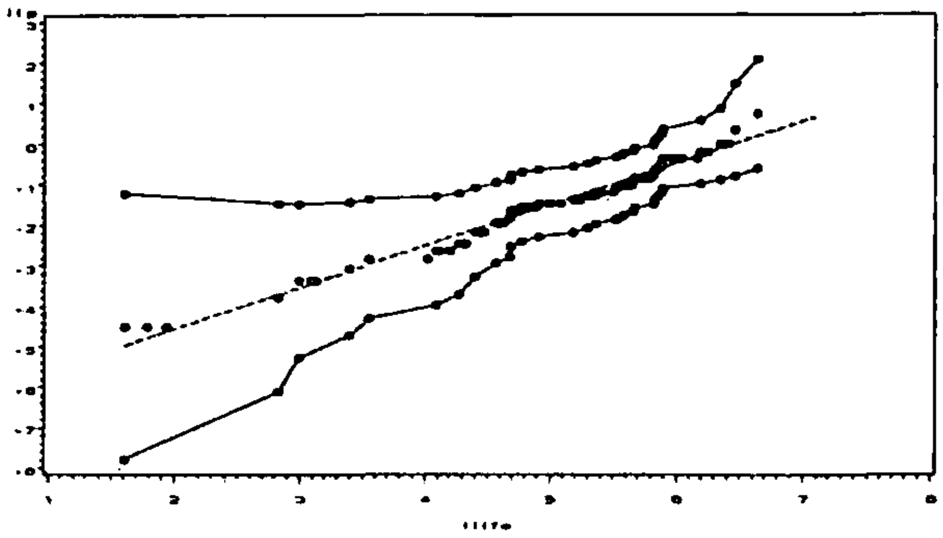

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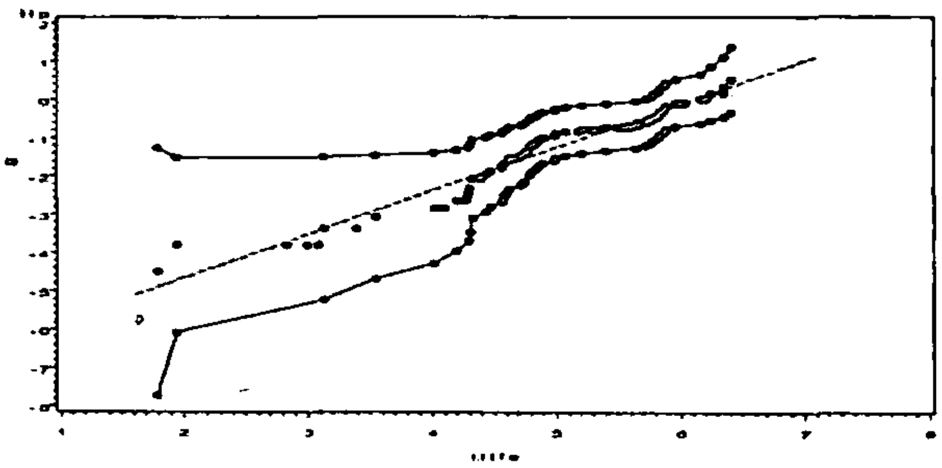

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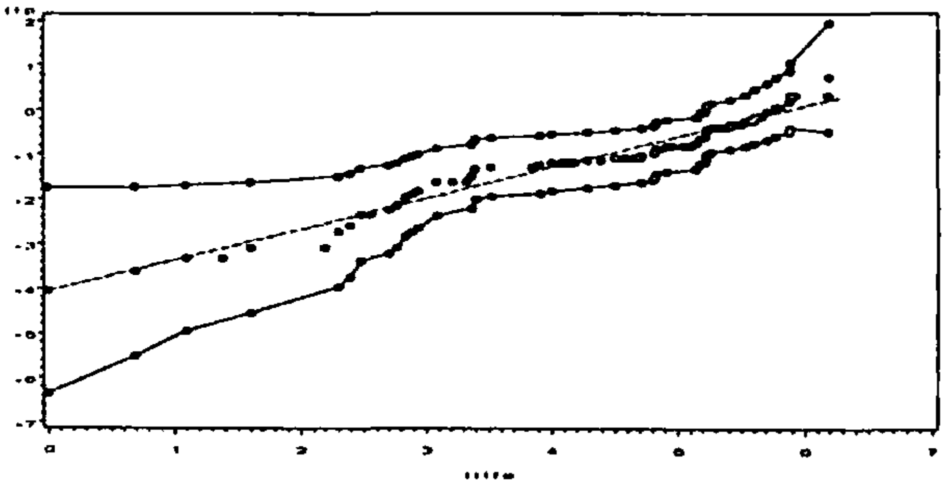

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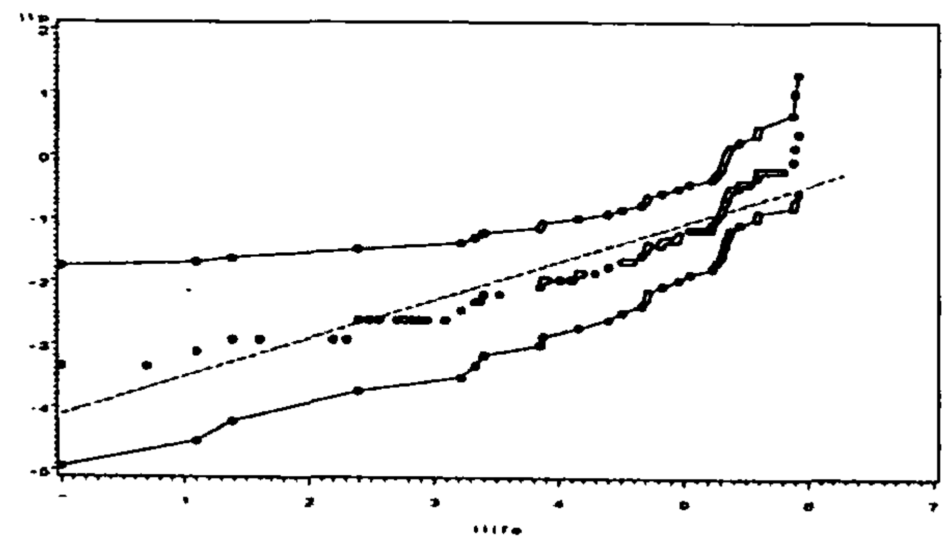

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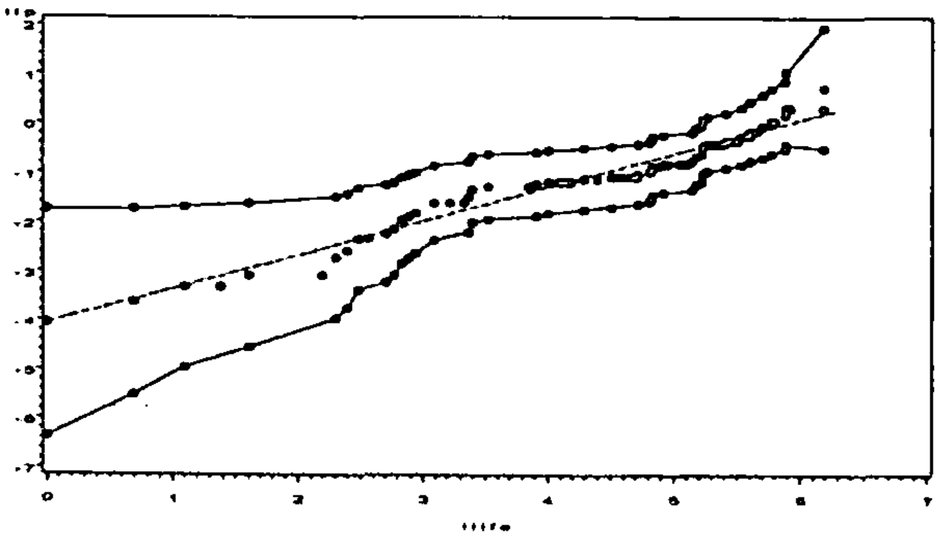

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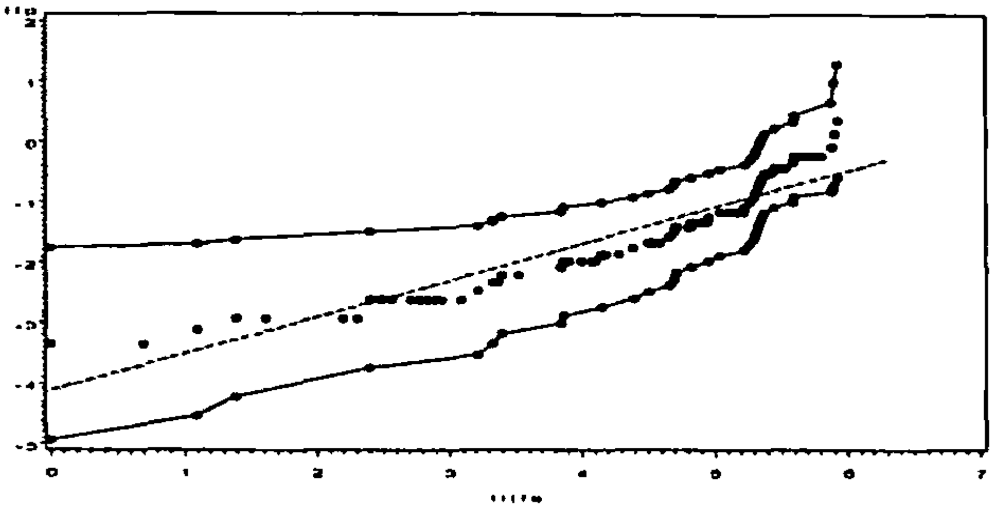

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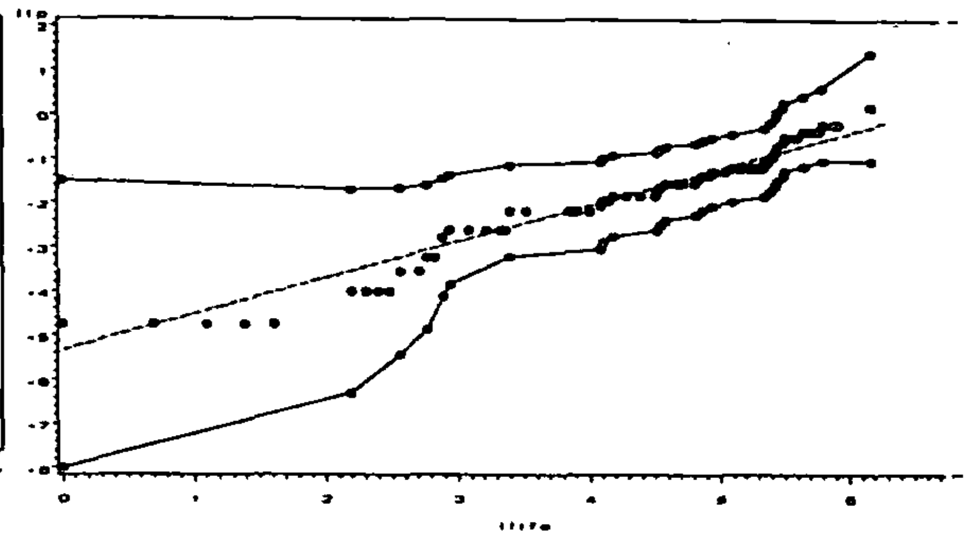

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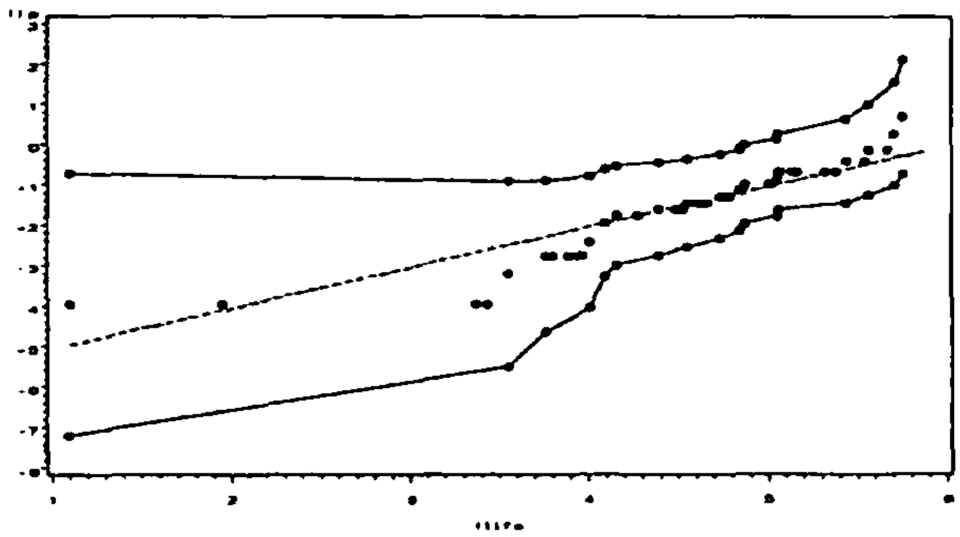

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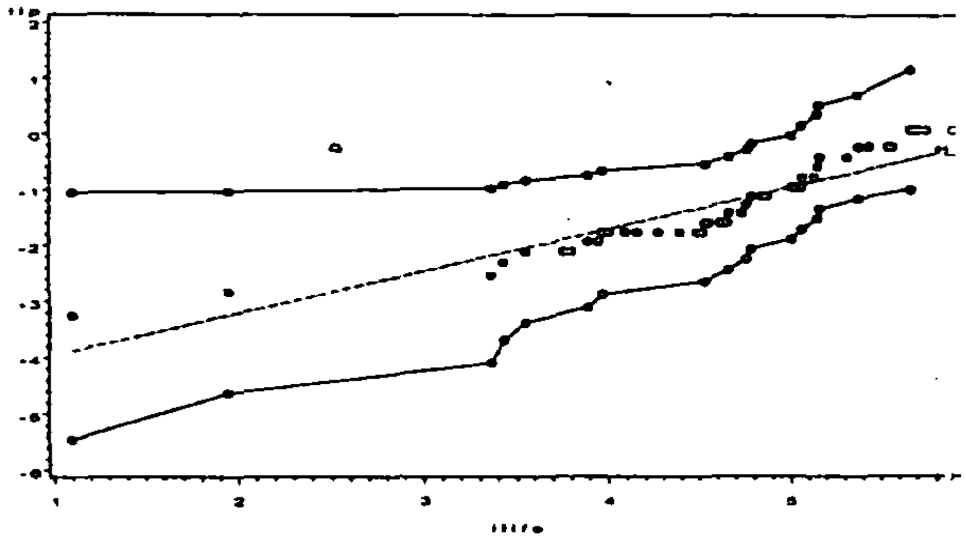

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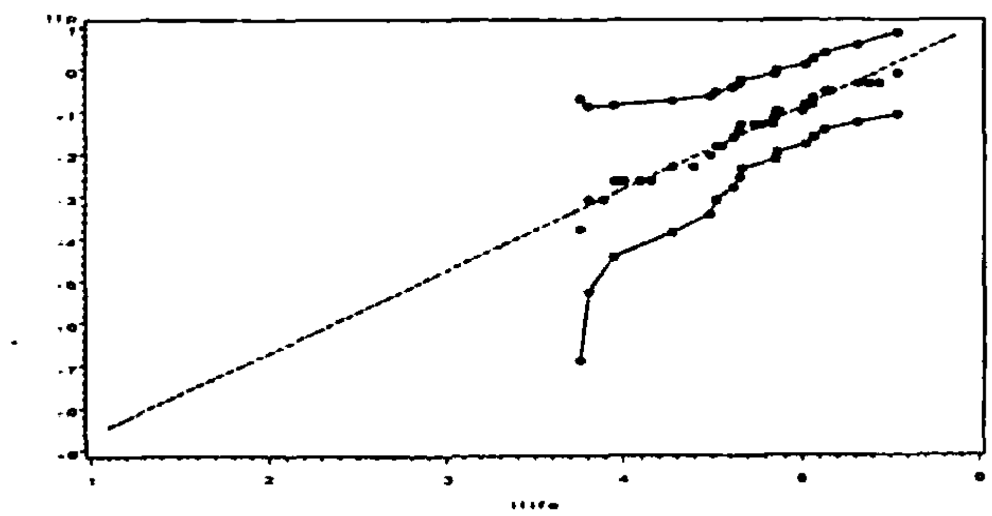

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Figure2: Weibull probability plots and simultaneous confidence bands by brand and type

Mann's goodness of fit test (Ebeling, 1997) was carried out to further strengthen the fact that each failure type of the five mobile phone brands is distributed as Weibull. The test statistics and the p-values of the tests for each failure type of each brand are given in the Table 3.

Table 3: Summary of Mann's goodness-of-fit test statistics and P-values

Brand	Type 1		Type 2		Type 3	
	Test Statistic	P-value	Test Statistic	P-value	Test Statistic	P-value
Brand 1	0.71912	0.91173	0.63350	0.98515	0.83881	0.76142
Brand 2	0.73500	0.98344	0.47870	0.99997	0.56366	0.99782
Brand 3	1.08093	0.41357	0.62412	0.80938	1.65246	0.05532
Brand 4	1.20549	0.25703	0.44902	0.98952	0.92003	0.58625
Brand 5	0.93830	0.54754	0.53861	0.90053	1.76096	0.13862

All the p-values of the Mann's goodness-of-fit tests were greater than 0.05. This suggests that the test is not significant (at the 5% level) for any of the failure types of the five brands and it can be concluded that each failure type has an approximate weibull distribution.

Modelling Failure Data with Multiple Modes of Failure

It was identified that each failure type of each brand follows a Weibull distribution. When fitting models for one failure type the other failure modes of that particular brand were assumed as censored. The likelihood ratio test was used to select between One weibull model for all brands (model 1), One weibull model with brand as a factor (model 2) and Separate weibull models for each brand (model 3). Table 4 consists of the likelihood ratio test statistic (LRTS), associated degrees of freedom and p value for comparing model 1 with model 2 and model 2 with model 3 for the three failure types separately.

The test Model 1 Vs Model 2 was significant for all three failure types as the test statistics were significantly greater than the critical value 9.48773, suggesting that the null model was significantly worse than fitting a model with brand as a factor for each failure type. The test Model 2 Vs Model 3 was highly significant for Type 1 and Type 2 suggesting that separate weibull models would model Type 1 and Type 2 failure data well compared to a model with brand as a factor. For type 3, the test Model 2 Vs Model 3 was not significant at the usual 5% significance level. The observed p-value was 0.1451, which is significant at 15% significance level. Therefore for the simplicity of parameterization and for convenience separate Weibull models were fitted to each failure type of five brands.

Table 4: Model comparison test statistics and degrees of freedom

Test	Type 1		Type 2		Type 3	
	LRTS	d.f.	LRTS	d.f.	LR TS	d.f.
Model: 1 Vs 2	16.28	4	27.25	4	19.83	4
Model: 2 Vs 3	28.06	4	10.78	4	6.83	4

The critical value was $\chi^2_{4, 5\%}$ (9.48773).

To check the adequacy of fitted Weibull models graphically, the Standardized Cox-Snell residuals were plotted (but not included in the paper) on Weibull probability plots. All the plots exhibit approximate linearity and fall within the simultaneous confidence bands. The intercepts range from -0.41178 to -0.03315 and the slopes range from 0.6946 to 1.1402 indicating that the intercepts and slopes are fairly close to 0 and 1 respectively. This suggests the adequacy of the fitted separate Weibull models to each failure type of the five brands.

Determining Characteristics of the Mobile Phone Brands

Comparison of the reliabilities of each phone brand

The fitted Weibull model for the j^{th} failure type of i^{th} brand can be written as follows;

$$\log_e(t_p) = \hat{\mu}_{ij} + \log_e[-\log_e(1-p)]\,\hat{\sigma}_{ij}$$

To estimate μ_{ij} and σ_{ij} the Maximum Likelihood method was used. The procedure PROC LIFEREG in SAS was used to obtain parameter estimates, their variances and covariances.

The industry standard of warranty period for a mobile phone is one year. Therefore the reliabilities for failure types 1, 2 and 3 for each mobile phone brand were calculated using equation (3) and the reliability of the entire system for each brand was calculated using equation (4) for a one year period and the survival probabilities (reliabilities) obtained are presented in Table 5. This table also gives the 95% confidence limits for the system for each brand using equations (6)-(11).

Table 5: Summary of survival probabilities for a one year period

Brand	Type 1	Type 2	Type 3	System		
				Reliability	Lower limit	Upper limit
1	0.3986	0.5441	0.6434	0.1395	0.1103	0.1750
2	0.3904	0.5267	0.5871	0.1207	0.0975	0.1486
3	0.5513	0.7508	0.4712	0.1950	0.1375	0.2691
4	0.3393	0.4384	0.4650	0.0692	0.0399	0.1173
5	0.2299	0.3139	0.2104	0.0152	0.0034	0.0652

According to the reliabilities of each failure type, it can be identified that Type 1 failure is the most common failure type that could occur in mobile phone brands 1, 2 and 4. That is failures with minor severity are more possible for those brands and they could survive more severe failures with high reliability. For brands 3 and 5 the most prevalent failure type is Type 3, suggesting that the brands 3 and 5 are less reliable with respect to most severe failures. Brands 3 and 5 can handle failures of moderate severity with high reliability.

For the entire system mobile phone Brand 3 has the highest probability of survival compared to the other phone brands during its one year warranty period. Brand 1 and Brand 2 are in the same ranking with respect to their survival rates. The next best brand would be Brand 4. The mobile phone brand which has the least chance of survival within one year warranty period is Brand 5. The confidence limits indicate that Brands 1, 2 and 3 have higher system reliability than brands 4 and 5. However the system reliabilities calculated for all brands for a one year period are fairly low, indicating that none of the mobile phone brands are very reliable.

Model based percentiles

The percentiles for each brand were calculated by solving equation (5) using the bi-section method. The 5^{th}, 10^{th} and 20^{th} percentiles (in days) are presented in Table 6.

Table 6: 5^{th}, 10^{th} and 20^{th} Percentiles for the System

Percentile	Brand 1	Brand 2	Brand 3	Brand 4	Brand 5
5^{th}	24.20	15.42	19.05	3.09	13.32
10^{th}	41.47	28.66	35.54	7.48	23.75
20^{th}	72.61	54.48	67.76	18.70	42.72

The percentiles are significantly low for all the brands further showing the low reliability of the phone brands. But for the Brand 1 a warranty period of 72 days can be declared if the 20% margin of failure is acceptable. Brand 4 has the lowest values at each percentile.

Mean Time To Failure (MTTF)

The mean time to failures in days for each failure mode and the overall system for each brand was determined respectively by solving equations (12) and (13) numerically. The results are summarized as in Table 7.

Table 7: Mean Time To Failure

Brand	Type 1	Type 2	Type 3	Overall System
Brand 1	350.628	502.997	638.319	202.370
Brand 2	375.410	483.736	541.613	180.026
Brand 3	534.744	747.950	457.943	226.888
Brand 4	389.876	506.411	476.922	124.175
Brand 5	253.628	309.833	250.909	119.131

Type 2 failure mode of Brand 3 has the highest MTTF and Type 3 failure mode of Brand 5 has the lowest MTTF compared to all the failure types of the five brands. For brands 1 and 2, the Type 3 failure mode has the highest MTTF and Type 1 mode has the lowest MTTF. Type 2 has the largest average time to failure for the brands 3, 4 and 5. For brands 1 and 2 Type 1 failures could occur earlier than the other two failure modes as the MTTF of Type 1 is less than that of types 2 and 3. The MTTF of the overall system is highest for Brand 3 compared to other phone brands. Brand 1 has the next highest overall system MTTF. Brands 2, 4 and 5 have MTTF values in descending order respectively. Therefore the ranking of the mobile phone brands from best to worst according to the overall system MTTF is Brand 3, Brand 1, Brand 2, Brand 4 and Brand 5. Since MTTF is a measure of center of the failure time distribution, it is worthwhile to compare MTTF with median which is also a measure of central tendency. Table 8 consists of the medians measured for each failure type and for the overall system of each brand.

Table 8: Median time to failure

Brand	**Type 1**	**Type 2**	**Type 3**	**Overall System**
Brand 1	300.296	404.686	518.693	168.287
Brand 2	274.979	388.352	445.651	142.170
Brand 3	415.104	652.224	338.572	176.583
Brand 4	202.665	296.371	330.434	72.832
Brand 5	217.011	225.875	226.773	99.010

From Table 8 it is observed that for all the failure types of all brands the MTTF is higher than its respective median. This is mainly due to the positive skewness of the failure data. For positively skewed distributions the mean is always greater than the median. Since each failure type of each brand was distributed as weibull which is also a positively skewed distribution, this observation is inevitable. Type 2 of Brand 3 has the largest median just as it has the largest MTTF. But Type 1 of Brand 4 has the smallest median whereas Type 3 of Brand 5 had the smallest MTTF value. For brands 1, 2, 4 and5, the Type 3 failure mode has the largest median and Type 1 mode has the smallest median. Type 2 has the largest median time to failure for the Brand 3. The ordering of the brands according to overall system medians which is Brand 3, Brand 1, Brand 2, Brand 5 and Brand 4 is some what different from the ordering under the overall system MTTF which is Brand 3, Brand 1, Brand 2, Brand 4 and Brand 5.

CONCLUSIONS

The main objective of this study is to examine and further develop the available statistical methods for analyzing multiple failure data in the field of reliability engineering by way of an application. In section 2.2 the authors develop methods for determining failure probabilities, percentiles, confidence limits and mean time to failure (MTTF) for a series system assuming that for each level of an explanatory variable and for each failure mode the failure data follows a weibull distribution. Here it is assumed that the different failure modes are independent of each other and if one type of failure occurs the entire system fails. The methods thus developed could be used for comparing the different levels of a factor with respect to reliability, for a series system experiencing multiple failures where the failure times follow a weibull distribution. These results could be easily generalized to several explanatory variables and other distributions.

A secondary objective of this study was to compare the reliability of different brands of mobile phones experiencing one of 3 types of failure. Separate Weibull models were fitted to each of the three failure modes of the five phone brands. According to overall system reliabilities and MTTF brands 1, 2 and 3 can be classified as "more reliable" phone brands compared to brands 4 and 5 which are less reliable. On average, users of mobile phone brands 1, 2, 3, 4 and 5 experience a failure every 202, 180, 226, 124 and 119 days respectively. The most prevalent failure types for brands 1, 2, 3, 4 and 5 are Type 1, Type 1, Type 3, Type 1 and Type 3 respectively where Type 1 indicates minor severity failures while Type 3 implies most serious/severe failures. From the percentiles and system reliabilities it can be concluded that one year warranty period is not worth for any of the brands considered in this study. Finally, Brand 3 could be identified as the best brand among the five brands considered in this analysis and other brands could be ranked as best in the order; Brand 1, Brand 2, Brand 4 and Brand 5.

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