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Using Cointegration for Stock Market Investment: An Application in a Sri Lankan Context

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ABSTRACT

This paper aims to incorporate previous research approaches of cointegration-based index tracking portfolio construction and adapt it to be suitable for a Sri Lankan application in the Colombo Stock Exchange (CSE). The All Share Price Index (ASPI) is selected as the benchmark index to be tracked. The paper explores an approach of index tracking to build a portfolio of sectors rather than stocks, while aiming to find the optimally cointegrated number of sectors and the best rebalancing frequency for the constructed portfolios. As a novelty, this paper introduces an approach to handle a constraint of all positive weights in the constructed portfolios. The results show that the designed portfolios are strongly cointegrated with the benchmark ASPI and indeed demonstrate good tracking performance. The results further show that all tracking sector portfolios constructed perform better than the ASPI for the same duration. However, for the study period considered, given the negative return levels of the CSE and all portfolios, a risk free Treasury bill investment can be considered as a better investment alternative than the CSE. The optimal level of cointegration was achieved with nine sectors in the portfolio. The best performing portfolio involved investing in the nine sectors of 'Banks, Finance and Insurance', 'Beverage, Food and Tobacco', 'Diversified Holdings', 'Healthcare', 'Hotels and Travels', 'Land and Property', 'Manufacturing', 'Plantations' and 'Telecommunications' with the portfolio weights rebalanced on an annual basis.

Keywords: Cointegration, Constrained allocations, Index tracking, Portfolio optimization

INTRODUCTION

Cointegration has many applications in econometric data. Some of which are its use in identifying long term relationships between macroeconomic factors, between financial markets and in portfolio construction. Its use in portfolio construction relates to investment allocation which has seen much of its development over the past decade owing to the work of Lucas (1997) and Alexander (1999). Stock Market investment can be singled out as a case in investment allocation. From the two approaches of Active and Passive strategies available to any stock market investor, Passive strategies which involve tracking portfolios are commonly dealt with using a cointegration approach. This paper will focus on such a tracking portfolio which tracks the main price index of the considered stock market.

Such tracking portfolios using cointegration theories have gained much popularity with fund managers the world over, even though its presence in Sri Lanka is yet to be established, since there is no published literature by local authors concerning this application. A lack of local research studies on stock portfolios could be attributed to the Colombo Stock Exchange (CSE) exhibiting weak form efficiency in comparison with other stock markets of the region (Gunasinghe, 2004).

The literature on portfolio allocation using cointegration are numerous and varied in their approaches. At the outset, Alexander (1999) focuses on introducing the concept of index tracking using an inter-regional Morgan Stanley Capital International (MSCI) Index. The paper explores the technique of heat maps in identifying the most cointegrating portfolio while focusing extensively on introducing the theory behind using cointegration for portfolio construction.

In comparison, diBartolomeo (1999) uses functions of surplus and liability together with Monte Carlo simulations to formulate his cointegrating portfolio which is previously not introduced under Alexander (1999). However, diBartolomeo (1999) does not rebalance[1] the cointegration relationship during all of the 14 years of testing. This problem is overcome in Alexander and Dimitriu (2002), where rebalancing is introduced at every ten trading days. As an improvement to previous studies this paper considers different stock selection criteria and focuses heavily on back-testing the constructed portfolios. Yet another different approach to stock selection is presented by Gabriel (2002) using an algorithm: where all possible stocks are considered initially and the stocks which contribute least to the cointegration are dropped one by one, leaving the optimally cointegrated portfolio of stocks. The rebalancing technique used here compares monthly rebalancing against unmanaged portfolios and follows similar back-testing criteria to Alexander and Dimitriu (2002).

[1] Rebalancing refers to re-estimating the weights making up the investment portfolio on a periodic basis. Not rebalancing the portfolio for the entire period of investing is called leaving the portfolio 'unmanaged'.

Dunis and Ho (2005) continuing similarly, introduces different rebalancing strategies of monthly, quarterly, semi-annually and annually in order to obtain the optimal rebalancing frequency. However the relatively simple stock selection criterion of using stock weights in the benchmark index is a considerable limitation of this paper compared to the many methods of Alexander and Dimitriu (2002).

Most studies discussed above are applications of portfolio construction relating to Western stock markets and deal with tracking either international (for example; MSCI in Alexander, 1999; diBartolomeo, 1999), industrial (Dow Jones Industrial Average in Alexander and Dimitriu, 2002) or regional (Dow Jones EUROStoxx50 in Dunis and Ho, 2005) price indices which combine stock indices across markets. As suggested by Alexander (2001), the complete index tracking investment strategy can be approached by constructing stock portfolios that track the respective sector index under each of the sectors included in the sector portfolio. However, few or no published studies have been undertaken in this regard internationally. In addition, the South Asian region does not account for a significant stake in the published literature on tracking portfolios as a whole.

Therefore, the aim of this paper is to incorporate two of the less frequent aspects mentioned above: by (i) constructing a portfolio of sectors rather than stocks and (ii) constructing the portfolios for the local stock market; the CSE. This paper aims to cover the features of tracking portfolios most commonly discussed in the above reviewed literature, including finding the optimally cointegrating portfolio and the best rebalancing frequency for a portfolio. In addition, as a novelty, this paper aims to handle the problem of constrained allocations which is previously not dealt with under the reviewed literature. Such a constrain is considered in this study because the CSE imposes a restriction on Short Sales[2], which translates to restricting the portfolio weights to be positive only (Alexander, 2001).

The remainder of this paper is organised as follows: the second section will highlight the methodology used for the portfolio construction process. The third section will discuss some important results of the study. The fourth section provides the conclusions and recommendations for further studies based on this application.

[2] Short Selling describes a trading strategy. This involves the sale of a security which is not owned by the seller, but that is promised to be delivered (www.investorpedia.com).

MATERIALS AND METHODS

This study constructs a portfolio using the 20 business sectors of the CSE in order to track the main price index of the CSE: the All Share Price Index (ASPI). The sector indices were specifically selected since the aim of this study is to build a portfolio of sectors. However this study could be extended so that the sector portfolio could be used to build stock portfolios that track the particular sector indices appearing in the sector portfolio. The data used for the study are the daily closing prices of the ASPI and its 20 constituent sector indices, for a five year duration beginning January 2004 and ending December 2008. The data set consists of 1,197 records of observations in total.

The methodology used in the present study were organised in three stages

i. Identifying the order of the data series

ii. Constructing the unmanaged portfolios and back-testing them

iii. Constructing the rebalanced portfolios and back-testing them

Identifying the Order of Integration of the Data Series

Prior to beginning the analysis, all series under consideration (the 20 sector indices and the ASPI) were transformed to their natural logarithms as it is conventionally done with financial data. This was done to produce more homogenous series, provided that, if the level variables are cointegrated, so will be their logarithms (Alexander and Dimitriu, 2002). As the cointegration theory states, the cointegration regression can only be applied to time series that are non-stationary or $I(1)$. Therefore, to determine if all series under consideration satisfied this requirement, tests for the presence of a unit root were carried out on all series using either Augmented Dickey-Fuller (ADF) tests or Phillip-Perron (PP) tests. The choice of using either the ADF or PP test is decided by the presence of a deterministic trend in the series.

For this study, a line graph of each *ln* transformed time series was inspected for deterministic trend. In instances of an apparent trend the PP was used, and in all other cases the ADF was used. However, both tests use the same ADF critical values. If the hypothesis of a unit root was rejected and the series concluded as stationary, such series were dropped from further analysis relating to cointegrating tracking portfolios in keeping with the cointegration theory.

Constructing the Unmanaged Portfolios

In the present study, the process of constructing the cointegrated portfolios under the unmanaged strategy was used as a base to build up on the rebalancing strategy comparison. Therefore, the methodology for building the unmanaged cointegrated portfolios is of significant importance. This approach is an adaption of the algorithm used for selecting the optimally cointegrated portfolio by Gabriel (2002). The adaption of the algorithm took into account conditions specific to the CSE such as the constrain on portfolio weights being positive (see Introduction). The unmanaged strategy considered using 15, 10, 5 and the optimal number of the most cointegrated sectors in the estimated Engle-Granger cointegrating regression model as was done under Alexander and Dimitriu (2002). Correlation was the classical approach to portfolio construction (Markowitz, 1952; 1959) where constructed tracking portfolios were required to show high levels of correlation with the benchmark. However, correlation analysis is valid only for stationary variables therefore making it necessary to de-trend the series (Alexander and Dimitriu, 2002). De-trending results in losing useful information which is present in the trend of a series. Using cointegration rather than correlation allows the use of the full information contained in the price indices, since no de-trending is required, and bases the portfolio weights on the long-run behaviour of the stocks (or sectors in our case).

Formulating the Cointegration Regression

For the remainder of the unmanaged portfolio methodology, the dataset was divided into two sections: in-sample and out-of-sample. The in-sample dataset was the first three years (1st January 2004 to 29th December 2006) and the out-of-sample (or test period) was the last two years (1st January 2007 to 31st December 2008). A three year in-sample period was considered since for a given number of stocks (sectors in our case) the returns tend to stay in the same range irrespective of the amount of historical data over three years used to determine the cointegration coefficients (Alexander and Dimitriu, 2002). Once the initial n number of $I(1)$ sectors were determined, the Engle-Granger cointegrating regression equation was estimated using these n sectors for the in-sample duration. In estimating the regression equation, the ASPI was taken to be the dependent variable and the predictor variables were selected from the n sector series.

If two or more series are themselves non-stationary, but a linear combination of them is stationary, then the series are said to be cointegrated (Engle and Granger, 1987). Testing for cointegration involves testing the residuals from an Ordinary Least Squares (OLS) regression for stationarity. In the context of the present study we estimated by OLS using the following equation:

$$\ln(ASPI_t) = \sum_{i=1}^{n} \beta_i . \ln(SECTOR_{i,t}) + \varepsilon_t \qquad (1)$$

where: $ASPI_t$ is the daily time series of benchmark index ASPI at time t; $SECTOR_{i,t}$ is the time series of sector i at time period t; β_i are the estimated coefficients from the above regression, which after normalization will play the role of portfolio weights; and ε_t is the residual series, which is the tracking error.

Testing the stationarity of the tracking error ε_t is done using the ADF test. Using *ln* prices has the advantage that the tracking error ε_t is in return format and the β_i coefficients are portfolio weights (Dunis and Ho, 2005).

Eliminating the least cointegrating sectors from the regression

Proceeding with the algorithm of Gabriel (2002); after finding the n number of $I(1)$ sectors, there should initially be n regression equations, each containing n-1 sectors as explanatory variables. We arrive at n-1 explanatory sectors by removing one explanatory sector at a time for each equation, in order to check for stationarity in the absence of the respective sector.

At the initial round (i=1), for each of the n regression equations, the respective residuals are extracted and the unit root tests is performed on them. Then if,

$$ADF_{1j} = \min \{ADF_{11}, ADF_{12}, \ldots ADF_{1n}\} \; ; \; 1 \leq j \leq n \qquad (2)$$

the sector corresponding to variable j is eliminated from the cointegration equation. After eliminating the minimum ADF valued sector, the second elimination round has n-1 regression equations, each containing n-2 explanatory sectors and so on for other elimination rounds. In general, the ADF test value of the extracted k^{th} series residual at the i^{th} elimination round is ADF_{ik}, $1 \leq i \leq 20$ and $1 \leq k \leq n$.

At each round of estimating the most cointegrating regression after eliminating a sector series, the non-negativity constraint corresponding to no Short Sales was checked; then those sectors showing negative coefficients were set to zero and the regression relationship was re-estimated as suggested by Alexander (2001).

For example, suppose at the initial round after eliminating the j^{th} variable sector, the following cointegration equation was obtained.

$$\ln(ASPI) = \beta_1 \ln(SECTOR_1) + \beta_2 \ln(SECTOR_2) + \ldots + \beta_{n-1} \ln(SECTOR_{n-1}) \qquad (3)$$

Here we retain only those β_i which satisfy $\beta_i > 0$. We set negative β coefficients (if any) to zero and re-estimate the regression equation such that all $\beta'_i > 0$ for the new cointegration equation having coefficients β'_i.

Finding the Optimally Cointegrating Number of Sectors

The process of eliminating sectors was continued up to a number m, where m took values of 15, 10, 5 and the optimally cointegrating number of sectors. The number m was varied so that we could compare between varying numbers of sectors in a portfolio and examine the performance of those portfolios against that of the ASPI as the number of sectors change. The optimally cointegrated regression of the above process was identified as the regression equation having the overall minimum residual ADF statistic from all ADF_{ij}'s from eliminating the j^{th} sector at the i^{th} round. Such a portfolio can consist of p sectors; where p could be 5, 10, 15 or even an in-between number depending on the relationship between the sector indices and ASPI that is tracked. Once the constrained β_i coefficients of the final four regression equations are normalised to sum to one, they act as portfolio weights. These weights were kept constant for the two-year out-of-sample period for the unmanaged strategy.

The performances of these unmanaged portfolios were then compared against the benchmark ASPI by using the back-testing methodology described earlier. This helped determine the best performing portfolio containing, say q (either: 5, 10, 15 or p) number of sectors.

Constructing the Rebalanced Portfolios

In order to compare different rebalancing strategies, we considered unmanaged (no rebalancing) versus (i) monthly rebalancing, (ii) semi-annual rebalancing and (iii) annual rebalancing strategies. Each rebalancing strategy assumed a fresh re-estimation of cointegration at each periodic time interval. These portfolios were then back-tested and their results compared against the ASPI and best performing q sector portfolio from the unmanaged strategy.

Dividing the sample data for rebalanced portfolios

For all strategies, the length of in-sample data remained constant at three years. In the case of the unmanaged strategy, the in-sample and out-of-sample lengths were three and two continuous years respectively, of the five year data. For the rebalanced strategies, the in-sample length was shifted periodically (monthly, semi-annually and annually), so that the cointegration regressions will be determined by using a fixed-length three-year shifting window of data immediately prior to the period for which the cointegrating portfolio was to be determined. That is, for the rebalancing strategy, the initial in-sample portfolio for the duration January 2004 to December 2006 was progressively shifted monthly, semi-annually or annually, while keeping a three-year window constant. This was continued until the final in-sample period ended prior to estimating the final cointegrating portfolio ending with 31st December 2008.

Therefore, while the unmanaged strategy estimated a single cointegrating portfolio of length two years, the rebalancing strategies resulted in estimating the following number of cointegrating portfolios.

Table 1: Summary of number of out-of-sample periodical portfolios

Rebalancing strategy	Number of periodical portfolios
Monthly	24
Semi-annual	4
Annual	2

The portfolios based on the four strategies above were compared using the back-testing criteria to select the best rebalancing frequency for the constructed sector portfolio.

Back-testing Methodology

The back-testing procedure considers approximately six criteria, which are described below, in order to assess the performance of the constructed portfolios. The same criteria were used in the unmanaged strategy where we were concerned with finding the best performing **number of sectors**, and in the rebalanced strategy where our focus was to find the best performing **rebalancing frequency**.

(i) Returns related measurements on the tracking portfolios were considered based on annualized daily returns of the ASPI and the tracking portfolios. If the portfolio weights for each k^{th} sector in the portfolio constructed at time period t is $w_{k,t}$ then the price of the portfolio after x days from the construction, $\lambda_{(t+x)}$, was calculated as follows.

$$\lambda_{(t+x)} = \sum_{k=1}^{m} w_{k,t} p_{k,(t+x)} \qquad (4)$$

where, t is the period at which the weights of the portfolio are determined

x is the number of days from t within one estimation cycle (using a 250 working-day year; $x < 20$ for monthly rebalancing, $x < 125$ for semi-annual rebalancing, $x < 250$ for annual rebalancing and $x < 500$ for unmanaged strategy)

$p_{k,(t+x)}$ is the daily price of the k^{th} sector index at x days after time t

m is the number of sectors in the portfolio taking values of 5, 10 and 15

The price of the portfolio can be simply considered as the weighted sum of the actual sector price indices in the portfolio. The daily portfolio returns were calculated as the first difference in *ln* portfolio prices at day t, that is, $\Delta \ln \lambda_t$; while the annualized returns of a portfolio were calculated as $365\Delta \ln \lambda_t$.

(ii) Daily excess returns (γ) were defined as simply the tracking errors of the portfolio.

$$\gamma = \Delta \ln \lambda_t - \Delta \ln \text{ASPI}_t \qquad (5)$$

And the annualized Daily excess returns, $\gamma_A = 365\gamma$ (6)

(iii) Using the convention of 250 working days per year, the annualized volatility (σ_{AV}) of daily portfolio returns was calculated as,

$$\sigma_{AV} = \sigma_{\Delta \ln \lambda_t}\sqrt{250} \qquad (7)$$

where, $\sigma_{\Delta \ln \lambda_t}$ is the standard deviation of $\Delta \ln \lambda_t$

Annualized volatility of daily excess returns were defined similarly considering σ_γ. Here we consider 250 days over 365 days since, while returns accumulate over the period for which the investment is held (365 days per year), the volatility of the portfolio price will take place only over the days which the market operates (250 days per year).

(iv) For each tracking portfolio constructed, the correlation of portfolio returns with excess returns with the benchmark returns were computed. The measurement for correlation was the Pearson Product-Moment Correlation coefficient (r).

(v) The Sharpe ratio (S) was computed as,

$$S = \frac{\bar{\gamma} - r_f}{\sigma} \tag{8}$$

where, γ is the average daily excess returns

r_f is the risk-free rate for the period under consideration

This paper considered r_f as the average interest rate of the three-month government Treasury bill (T-bill) rate in Sri Lanka over the sample five year period, which was 11.8% per annum (www.cbsl.gov.lk).

(vi) The Information ratio (I) used in this paper is that defined according to Alexander and Dimitriu (2002). This differs from the Sharpe ratio by not including the risk-free rate.

$$I = \bar{\gamma} / \sigma \tag{9}$$

As shown in the literature of portfolio construction (Dunis and Ho, 2005; Alexander and Dimitriu, 2002), the best performance of a particular portfolio is based on its Sharpe and Information ratios. Therefore under both instances (selecting the best number of sectors and best rebalancing frequency), the portfolio which shows the largest Sharpe and Information ratios was considered as the best. To support the decision from these two ratios, sensible scrutiny of the ratios as well as the best performance in the majority of the other measurements was used.

RESULTS AND DISCUSSION

The preliminary analysis concluded that five of the 20 business sectors of the CSE were stationary time series, leaving only 15 $I(1)$ sectors suitable for further analysis. These stationary sectors were 'Footwear and Textile', 'Investment Trusts', 'Oil Palms', 'Services' and 'Stores and Supplies'. The results of the unit root tests for all 20 sector series can be found in Appendix A. The existence of stationary sector indices could be attributed to those sectors being in their preliminary stages. Financial data series as a convention are expected to follow an $I(1)$ pattern. However, the above five sector series being $I(0)$ indicate that their trends over time can be determined and is in agreement with the CSE showing weak form efficiency (Gunasinghe, 2004).

For the initial 15 $I(1)$ sectors, the resulting constrained equation however included only 12 sector series, even though we set out to achieve a portfolio with 15 cointegrated sectors. The unmanaged strategy showed that having nine sectors in the tracking portfolio gave the optimal level of cointegration. The cointegrating tracking portfolios constructed under the unmanaged strategy can be found in Table 2 below.

Apart from the five stationary sector series mentioned previously, it is interesting to note that the sectors of 'Chemicals and Pharmaceuticals', 'Construction and Engineering' and 'Information Technology' are not featured in any of the four portfolios constructed under the unmanaged strategy. This could be attributed to the fact that these sectors were recently started or have only few companies that come under them and they do not account for much of the market capitalisation of the CSE. For example, the 'Information Technology' sector which began in May 2002 and consists of one company only up to date has a market capitalization of only 0.02% of the CSE.

Table 2: Constrained and normalized weights of portfolios constructed for the unmanaged strategy

	Number of sectors in portfolio			
	5	**9**	**10**	**12**
Banks Finance and Insurance	31.51%	24.53%	24.35%	24.27%
Beverage, Food and Tobacco	-	11.00%	10.40%	10.88%
Chemicals and Pharmaceuticals	-	-	-	-
Construction and Engineering	-	-	-	-
Diversified	42.59%	28.64%	28.52%	28.12%
Footwear and Textile	-	-	-	-
Healthcare	-	1.42%	1.71%	1.48%
Hotels and Travels	5.57%	10.30%	10.23%	10.56%
Investment Trusts	-	-	-	-
Information Technology	-	-	-	-
Land and Property	-	1.02%	0.82%	0.63%
Manufacturing	3.41%	10.91%	10.52%	8.75%
Motors	-	-	-	0.69%
Oil Palms	-	-	-	-
Plantations	16.92%	5.57%	5.36%	5.31%
Power and Energy	-	-	-	0.75%
Services	-	-	-	-
Stores and Supplies	-	-	-	-
Telecommunications	-	6.62%	6.97%	6.79%
Trading	-	-	1.14%	1.77%

When considering the back-testing criteria for the unmanaged strategy, all four constructed portfolios showed higher returns than the ASPI's annualized return of -46.01% over the out-of-sample period, even though still maintaining negative return levels. The out-of sample period considered for the unmanaged strategy was a particularly volatile two years for the ASPI. However, due to their cointegrating relationship, even in such a circumstance, the portfolios out-perform the benchmark when left unmanaged. The results for all back-testing criteria used to measure the performance of the unmanaged portfolios are given in Table 3.

Table 3: Summary back-testing results for out-of-sample unmanaged portfolio performance

Portfolio	Average Annualized Portfolio Returns	Annualized Volatility of Portfolio Returns	Average daily Excess Returns	Average Annualized Excess Returns	Annualized Volatility of Excess Returns	Correlation of Returns with ASPI Returns	Correlation of Excess Returns with ASPI Returns	Sharpe Ratio	Information Ratio
Benchmark (ASPI)	-46.01%	15.68%	-	-	-	1.0000	-	-3.687	-2.934
12 Sectors	-36.56%	14.37%	0.0259%	9.45%	6.66%	0.9054	-0.4004	-3.365	-2.544
10 Sectors	-36.86%	14.42%	0.0251%	9.15%	6.69%	0.9045	-0.3934	-3.374	-2.555
9 Sectors (OPTIMAL)	-36.42%	14.35%	0.0263%	9.59%	6.76%	0.9024	-0.4035	-3.360	-2.538
5 Sectors	-40.70%	14.86%	0.0146%	5.31%	7.70%	0.8742	-0.3489	-3.532	-2.738

From these four portfolios, the nine sector portfolio shows the best returns-performance by having comparatively higher returns and lower volatilities than other portfolios and the ASPI. It also posted the best Sharpe ratio of -3.36 from all unmanaged portfolios, against the ASPI's -3.687 for the same period.

The literature (Gabriel, 2002; Dunis and Ho, 2005) rebalances the best performing unmanaged tracking portfolio. This was attempted and proved unsuccessful in the present context due to the non-negativity constraint causing in very few sectors to finally remain in the portfolios. Therefore each rebalancing assumed fresh re-estimations of cointegration and constraining of weights (in the same way as the unmanaged strategy). The portfolios obtained in this way under all three rebalanced strategies, ended with at most 9 sectors. Since under the unmanaged strategy, we previously found the nine sector portfolio to perform best, the rebalanced strategies were also limited to a maximum of nine sectors. All portfolios constructed under the three rebalancing strategies of monthly, semi-annual and annual gave residual ADF statistics which showed stationarity at 5% level, implying that the cointegration relationship was valid for all constructed portfolios. These results for the rebalanced strategy can be found in Appendix B.

The portfolios under all three rebalancing strategies most frequently contained the six sectors of 'Banks, Finance and Insurance', 'Beverage, Food and Tobacco', 'Diversified Holdings', 'Hotels and Travels', 'Manufacturing' and 'Telecommunications'. Upon further investigation, these six sectors were also found to be the dominant sectors of the CSE and thereby the most significant to invest in.

All rebalanced portfolios constructed showed better performance than both the ASPI and the unmanaged strategy even though still maintaining negative returns. The results for all back-testing criteria used to measure the performance of the rebalanced portfolios are presented in Table 4.

Table 4: Summary back-testing results for out-of-sample rebalanced portfolio performance

Portfolio	Average Annualized Portfolio Returns	Annualized Volatility of Portfolio Returns	Average daily Excess Returns	Average Annualized Excess Returns	Annualized Volatility of Excess Returns	Correlation of Returns with ASPI Returns	Correlation of Excess Returns with ASPI Returns	Sharpe Ratio	Information Ratio
Benchmark (ASPI)	-46.01%	15.68%	-	-	-	1.0000	-	-3.687	-2.934
Unmanaged Optimal 9 Sector	-36.42%	14.35%	0.026%	9.59%	6.76%	0.9024	-0.4035	-3.360	-2.538
Monthly Rebalancing	-34.38%	72.00%	0.0319%	11.63%	71.14%	0.1636	-0.0548	-0.641	-0.477
Semi-annually Rebalancing	-35.47%	17.40%	0.0289%	10.53%	11.07%	0.7806	-0.1895	-2.718	-2.039
Annually Rebalancing	-28.06%	16.27%	0.0492%	17.95%	9.52%	0.8232	-0.2402	-2.450	-1.725

The annually rebalanced strategy scores well in terms of Annualized Portfolio Returns and maintains mid-range volatility values compared to other rebalancing strategies. The best Sharpe ratio of -0.641 recorded by the monthly rebalanced strategy is somewhat misleading due to its very high volatility. The unusually high 72% volatility for the monthly rebalancing is due to some significant sectors being left out of the monthly portfolios at different monthly periods. Therefore, after neglecting the Sharpe ratio of the monthly rebalancing the next highest ratio of -2.45 for annual rebalancing was considered as the best Sharpe ratio.

The costs associated with transactions for the rebalanced strategies have not been considered in this paper. However, the results of this study indicate that the best performing rebalancing strategy would remain unchanged even if this fact was considered, since annual rebalancing occurs the least frequent from all considered rebalancing strategies and thereby incurs the least total transaction cost on rebalancing.

Given that all Sharpe ratios calculated under both unmanaged and rebalanced strategies were negative, it is reasonable to conclude that risk free investments (in this case three-month Sri Lankan Government T-bills), would be a better investment for the considered period than stock market investment. However, since all portfolios outperformed the ASPI even when the market was on a downturn, it implies that when the market improves, the constructed cointegrating portfolios can be expected to generate good returns. Possible future upward market trends could result in positive Sharpe ratios for the constructed portfolios, thereby making investing in the CSE more attractive to investors than the risk free T-bills as the results currently indicate.

It should be noted that the performance results obtained are heavily dependent on the data of the out-of-sample period, which translates to varying time periods from years 2007 to 2008 for all strategies. When exploring the market conditions that prevailed during these periods, the CSE showed sensitivity to the local political climate and international economic conditions. The key to a successful application of cointegration in portfolio construction is to have a market that is relatively stable and mature, so that it is reasonable to assume that the trends in the past will continue for the foreseeable future. Therefore, while the return levels of the proposed portfolios may not seem appealing for investing in the CSE right now, it does demonstrate good tracking and better performance over the benchmark; which means this methodology is timely and valid for the CSE and can always be used when the market situation stabilises over time.

CONCLUSIONS AND RECOMMENDATIONS

To ensure optimal cointegration in a tracking portfolio of the CSE, the number of sectors to hold in a portfolio is nine. The same number of sectors (nine) showed the best performance when the portfolio was left unmanaged for the two year test period. The optimal rebalancing frequency of such a sector portfolio in the CSE was on an annual basis. During the study period, the most significant or dominant sectors in the CSE to invest in were; 'Banks, Finance and Insurance', 'Beverage, Food and Tobacco', 'Diversified Holdings', 'Hotels and Travels', 'Manufacturing' and 'Telecommunications'.

For the considered out-of-sample period, the CSE generated negative Sharpe ratios, indicating that the selected risk-free investment of three-month Sri Lankan Government T-bill to be more lucrative to invest in.

In addition to being used for investing in the stock market, the constructed sector portfolios can also serve as a guide to any investor looking to diversify his investments outside the stock market, into various industrial sectors.

Going beyond a sector portfolio and exploring the full investing strategy into particular stocks of the CSE will be useful for investors and therefore can be considered as a future expansion of the present study. For future applications, it would be best if the element of transaction costs be incorporated, since it can affect the cost involved in rebalancing more frequently. In addition, building estimation models to model the behaviour of the volatility of the constructed portfolios could also be a suitable direction for later research.

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Appendix A: Unit root tests for all series in dataset (1st January 2004 – 30th December 2008)

Series name	ADF level	Probability	ADF 1st diference	Probability	PP level	Probability	PP 1st diference	Probability	Conclusion
LASPI	-2.0028	0.2858	-28.6368	0.0000	-2.2346	0.1942	-28.4987	0.0000	$I(1)$
LBANKS	-1.2836	0.6391	-29.8993	0.0000	-1.3926	0.5872	-29.7584	0.0000	$I(1)$
LBEV	-1.6751	0.4437	-30.3773	0.0000	-1.8194	0.3713	-30.1899	0.0000	$I(1)$
LCHEM	-2.7663	0.0635	-33.9518	0.0000	-2.7767	0.0619	-33.9797	0.0000	$I(1)$
LCONSTR	-0.9896	0.7588	-27.9112	0.0000	-1.0444	0.7390	-37.6727	0.0000	$I(1)$
LDIV	-1.8215	0.3702	-30.2568	0.0000	-2.0166	0.2798	-30.2213	0.0000	$I(1)$
LFOOT	-3.3332	0.0137	-32.5978	0.0000	-3.3488	0.0131	-32.6525	0.0000	$I(0)$
LHEALTH	-1.9543	0.3074	-33.0963	0.0000	-1.9533	0.3079	-33.0634	0.0000	$I(1)$
LHOTELS	-2.7277	0.0696	-37.2102	0.0000	-2.9501	0.0401	-37.4926	0.0000	$I(1)$
LINV	-3.9922	0.0015	-32.5929	0.0000	-3.9791	0.0016	-32.5419	0.0000	$I(0)$
LIT	-2.3975	0.1427	-28.4306	0.0000	-2.2054	0.2046	-42.1987	0.0001	$I(1)$
LLAND	-2.0374	0.2708	-30.6726	0.0000	-2.0857	0.2506	-30.4644	0.0000	$I(1)$
LMANUF	-2.2692	0.1823	-30.0144	0.0000	-2.4557	0.1268	-29.9471	0.0000	$I(1)$
LMOT	-1.8975	0.3337	-33.8383	0.0000	-1.8881	0.3381	-33.8361	0.0000	$I(1)$
LOILP	-4.4267	0.0003	-42.0895	0.0000	-4.4116	0.0003	-45.2090	0.0001	$I(0)$
LPLANT	-1.5071	0.5299	-30.1033	0.0000	-1.5126	0.5271	-30.3300	0.0000	$I(1)$
LPOWER	-1.1406	0.7015	-26.9672	0.0000	-1.1600	0.6935	-34.0261	0.0000	$I(1)$
LSERV	-3.0909	0.0275	-27.2533	0.0000	-2.8112	0.0570	-35.4192	0.0000	$I(0)$
LSTORES	-6.7073	0.0000	-34.3071	0.0000	-6.7326	0.0000	-34.3071	0.0000	$I(0)$
LTELECOM	-0.8381	0.8074	-33.9228	0.0000	-0.8262	0.8109	-33.9229	0.0000	$I(1)$
LTRAD	-1.4929	0.5371	-33.0074	0.0000	-1.4784	0.5444	-32.9788	0.0000	$I(1)$

*MacKinnon critical values for rejection of hypothesis of a unit root

1% level	-3.43563
5% level	-2.86376
10% level	-2.56800

Appendix B: ADF statistics for rebalanced portfolios

Table B.1: Normalised final weights for monthly rebalanced portfolio

Sector	P_{m1}	P_{m2}	P_{m3}	P_{m4}	P_{m5}	P_{m6}	P_{m7}	P_{m8}	P_{m9}	P_{m10}	P_{m11}	P_{m12}
LBANKS	0.24525	0.23128	0.26853	-	-	0.24526	0.24919	0.26302	-	0.30387	-	0.28175
LBEV	0 10997	0.10407	0.13350	0.44261	0.34411	0.09759	0.09637	0.10439	0.13978	0.03987	0.13596	0.09507
LCHEM	-	-	0.05900	-	-	-	-	-	-	0.03777	0.00727	0.03318
LDIV	0.28643	0.23691	-	0.07938	0.19697	0.14963	0.14836	0.14638	0 18645	0.23141	0.18721	0.12822
LHOTELS	0.10304	0.10343	0.15637	0.17414	0.16038	0.12587	0.12205	0.11890	0.12985	-	0.11395	0.10288
LMANUF	0.10911	0.16647	0.13836	0.09363	0.17809	0.22503	0.22868	0.21618	0.18560	0.02107	0.13869	0.16983
LPLANT	0.05565	0.04692	-	0.01469	0.01178	0.03030	0.02871	0.02515	0.02620	0.02844	0.01038	-
LTELECOM	0.06617	0.09402	0.15480	0.08679	0.06491	0.12631	0.12664	0.12207	0.16347	0.11576	0.16584	0.12992
LTRAD	-	0.00735	-	-	-	-	-	-	0.16866	0.17380	0.23635	0.05081
ADF Statistic	-5.71491	-6.20503	-7.64371	-3.21326	-3.42034	-3.66999	-4.01904	-3.76199	-4.35526	-5.21561	-5.05638	-4.09108
Number of sectors	9	9	9	8	7	7	7	8	7	9	9	9

Sector	P_{m13}	P_{m14}	P_{m15}	P_{m16}	P_{m17}	P_{m18}	P_{m19}	P_{m20}	P_{m21}	P_{m22}	P_{m23}	P_{m24}
LBANKS	0.27144	0.28280	0.29696	0.31194	0.31156	0.31709	0.29091	0.28090	0.29157	0.29091	0.28090	0.29157
LBEV	0.08874	0.03308	0.00090	-	-	0.00131	0.02450	0.03964	0.04461	0.02450	0.03964	0.04461
LCHEM	0.03494	0.04919	0.05552	0.05300	0.04893	0.04554	0.03175	0.02743	0.02429	0.03175	0.02743	0.02429
LDIV	0.12544	0.11910	0.12214	0.13336	0.13532	0.13379	0.12601	0.12158	0.12383	0.12601	0.12158	0.12383
LHOTELS	0.11367	0.09761	0.08867	0.09098	0.09353	0.09792	0.12048	0.12801	0.12942	0.12048	0.12801	0.12942
LMANUF	0.19491	0.21140	0.20793	0.19423	0.19002	0.18498	0.16738	0.16029	0.15140	0.16738	0.16029	0.15140
LPLANT	-	-	-	-	-	-	0.00303	0.00255	0.00243	0.00303	0.00255	0.00243
LTELECOM	0.13396	0.15774	0.17112	0.17377	0.17849	0.18660	0.21002	0.21484	0.21586	0.21002	0.21484	0.21586
LTRAD	0.03691	0.04908	0.05677	0.04271	0.04215	0.03277	0.02592	0.02477	0.01658	0.02592	0.02477	0.01658
ADF Statistic	-3.92446	-3.99007	-4.16926	-3.92612	-3.98925	-4.22660	-4.34616	-4.00065	-3.93064	-4.34616	-4.00065	-3.93064
Number of sectors	8	8	8	7	7	8	9	9	9	9	9	9

Table B.2: Normalised final weights for semi-annually rebalanced portfolio

	P_{s1}	P_{s2}	P_{s3}	P_{s4}
LBANKS	0.24525	0.24919	0.27144	0.29091
LBEV	0.10997	0.09637	0.08874	0.02450
LCHEM	-	-	0.03494	0.03175
LCONSTR	-	-	-	-
LDIV	0.28643	0.14836	0.12544	0.12601
LHEALTH	0.01422	-	-	-
LHOTELS	0.10304	0.12205	0.11367	0.12048
LIT	-	-	-	-
LLAND	0.01016	-	-	-
LMANUF	0.10911	0.22868	0.19491	0.16738
LMOT	-	-	-	-
LPLANT	0.05565	0.02871	-	0.00303
LPOWER	-	-	-	-
LTELECOM	0.06617	0.12664	0.13396	0.21002
LTRAD	-	-	0.03691	0.02592
ADF Statistic	-5.714905	-4.019043	-3.924456	-4.346163
Number of sectors	9	7	8	9

Table B.3: Normalised final weights for annually rebalanced portfolio

	P_{a1}	P_{a2}
LBANKS	0.24525	0.27144
LBEV	0.10997	0.08874
LCHEM	-	0.03494
LCONSTR	-	-
LDIV	0.28643	0.12544
LHEALTH	0.01422	-
LHOTELS	0.10304	0.11367
LIT	-	-
LLAND	0.01016	-
LMANUF	0.10911	0.19491
LMOT	-	-
LPLANT	0.05565	-
LPOWER	-	-
LTELECOM	0.06617	0.13396
LTRAD	-	0.03691
ADF Statistic	-5.714905	-3.924456
Number of sectors	9	8