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PHYSICAL QUANTITIES AND UNITS

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Introduction

When we see a van travelling on a road we might say 'That van is travelling at about 85' or 'That van is travelling at 84.6 kilometers per hour'. The first statement gives information in an approximate form, acceptable in daily life and takes the units of measurements for granted. However, it is seldom good enough to use it for scientific purposes due to two reasons. It is not 'precise', and it is open to misunderstanding. For example, to a person in the United Kingdom it would mean '85 miles per hour' while to a person in Sri Lanka it would be presumed either as 'kilometers per hour' or 'miles per hour'. Hence, the second statement is much more informative and precise. The units of measurement are clearly stated and the speed is given with some 'precision'. Further, according to the second statement the van was not traveling either at '84.5 kilometers per hour' or at '84.7 kilometers per hour'. Therefore, students studying science or professionals conducting scientific investigations should report their quantities precisely and accurately with acceptable units and in an acceptable manner for the scientific community. The purpose of this chapter is to review the basic concepts related to documentation of results in scientific investigations.

Numbers, Physical properties and Quantities

Numbers are abstract symbols that, by themselves, have no real physical meaning. To use numbers in quantifying a physical quantity or entity, the dimensions of the unit quantity must be specified, and the number and its units must be treated as a single entity. Let us first see the properties and features of the physical quantities, briefly.

Basically a physical property is a measurable (or perceived) property of something observable without having to change the composition or the identity of that substance. Examples are: length, mass, colour, temperature, etc. These physical properties can be divided into the following subsets: mechanical, electrical, thermal and optical properties. A physical quantity is also a quantity that can be measured and expressed in numbers and units. Moreover, it can be represented by a symbol of the quantity, a numerical value for the magnitude of the quantity and the units of measurements.

In general there are two types of physical quantities. One is called 'scalar quantity', which has only a magnitude and no direction. For example time, temperature, mass, density, etc. The other is called 'vector quantity' which has both magnitude and direction. For example, displacement, velocity, forces, etc. However, to get a meaningful idea about the magnitude of these quantities, it is necessary to use some 'units' with these physical quantities. Therefore in the 1960s, the International System of units (SI) was established. With the introduction of SI units, all these physical quantities were categorized into two groups: 'basic (base or fundamental) quantities' and 'derived quantities'. These basic or fundamental quantities cannot be defined in terms of other base quantities. Examples of basic quantities are given in Table 1 with their SI unit name and symbol.

Table 1. Basic quantities with units and symbols

Basic quantity	Units name	Unit symbol
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	Ampere	A
Temperature	Kelvin	K
Luminous intensity	candela	cd
The amount of substances	mole	mol

The other category or derived quantities are defined as quantities, which are derived from fundamental or base quantities. Some examples are listed in Table 2, with the corresponding SI unit name and symbols.

Table 2. Derived quantities with units and symbols

Derived quantity	Units Name	Unit and Symbol
Area	Square meter	m ²
Volume	Cubic meter	m ³
Velocity	Meters per second	m/s
Force	Newton	N
Density	Kilogram per cubic meter	kg/m ³
Power	Watt	W

In science we generally deal with physical quantities and not with pure numbers as in mathematics. This means that virtually every number we write down should have units associated with it and depending on the units that we choose, the numerical value of a physical quantity will vary. For example, if we consider a

quantity of energy of 1 kJ, this could equally well be expressed as 1000 J, or 6.242×10^{21} eV. Hence, when dealing with physical quantities, just writing down a number without stating its units is completely meaningless. Therefore, a physical quantity can be expressed as the product of a numerical value and a unit:

$$(\text{Physical quantity}) = (\text{numerical value}) \times (\text{unit})$$

e.g. (weight of an average person) = (70) × (kg) and this is expressed as 70 kg.

All calculations involving units will essentially become basic algebra. For example, the tick marks along the axis of a graph are generally only labeled with numerical values. Therefore, the labeling of an axis must be consistent with the above expression. If we rearrange the above expression, then it can be rewritten as:

$$(\text{numerical value}) = (\text{physical quantity})/(\text{unit}),$$

and therefore, the axes should always be labeled to be consistent with this. For example, 'speed/ms⁻¹' or 'mass/kg'. This means that the speed is expressed in 'meters per second' and the mass is expressed in 'kilograms'. However, in some textbooks and scientific publications an alternative method, but technically incorrect, of expressing units of the graph such as 'speed (ms⁻¹)' or 'mass (kg)', can be found. Using parenthesis or brackets to indicate the unit of measurement of a physical quantity is now obsolete.

Usually, a single letter in the alphabet is used to represent a physical quantity and is often printed in italic type (*F*, *t*, *m*,...) and sometimes with modifications of subscripts and superscripts, to specify what they refer to. For example E_k is usually used to denote kinetic energy and C_v for heat capacity at constant volume. Table 3, shows some of the popular and commonly used examples. As a rule, in general, all symbols used in equations, figures, and tables should be defined upon their first use in any written document. If it is necessary to use the same symbol with two different meanings, both must be defined.

Table 3. Examples for usage of Greek letters for physical quantities with subscripts

Symbol	Description
C_p	Heat capacity at constant pressure
x_i	Mole fraction of the <i>i</i> th species
C_B	Heat capacity of substance B
E_k	Kinetic energy
μ_r	Relative permeability

Standard Scaling Prefixes

The size of a standard dimension can be changed by using a prefix, which indicates the relationship to the fundamental unit. Prefixes are very useful when dealing with very small or very large physical quantities. The most commonly used prefixes are listed in Table 4.

Table 4. Unit prefixes for power of 10

Prefix	Symbol	Multiple	Prefix	Symbol	Multiple
Exa	E	10^{18}	Deci	d	10^{-1}
Peta	P	10^{15}	Centi	c	10^{-2}
Tera	T	10^{12}	Milli	m	10^{-3}
Giga	G	10^9	Micro	μ	10^{-6}
Mega	M	10^6	Nano	n	10^{-9}
Kilo	k	10^3	Pico	p	10^{-12}
Hecto	h	10^2	Femto	f	10^{-15}
Deka	da	10^1	Atto	a	10^{-18}

Intensive and Extensive Quantities

Physical quantities can be further divided into two sub groups just by considering the amount of the matter present in that quantity. They are called 'intensive quantities' and 'extensive quantities'. An intensive quantity or property is a physical property of a system that does not depend on the size of the system or the amount of material in the system. Examples are: temperature, chemical potential, density, specific gravity, viscosity, velocity, and electrical resistivity. On the other hand, if a property of a system depends on the size of the system and (or) the amount of the material, such a property is called an extensive property. Examples are: mass, volume, entropy, enthalpy, stiffness, etc.

Expressing Values and Quantities

Value and Numerical value of a Physical quantity

As stated previously, the *value* of a physical quantity is its magnitude expressed as the product of a number and a unit. The number multiplied by the unit is the *numerical value* of the quantity expressed in that unit.

In a formal way, the value of quantity A can be written as $A = \{A\}[A]$, where $\{A\}$ is the numerical value of quantity A , when the value of it is expressed in the

unit $[A]$. Therefore, the numerical value can be written as $\{A\} = A / [A]$, which is a convenient form to use in figures and tables. Thus, to eliminate the possibility of misunderstanding, an axis of a graph or the heading of a column of a table can be labeled as ' $t/^\circ\text{C}$ ' instead of ' $t(^\circ\text{C})$ ' or 'Temperature ($^\circ\text{C}$)'. Similarly, an axis or column heading expressing the electric field strength can be labeled as ' $E/(\text{V/m})$ ' instead of ' $E(\text{V/m})$ ' or 'Electric field strength (V/m)'. Let us discuss a simple example for this, where the ordinate (or the y-axis) of a graph is labeled as $T/(10^3 \text{ K})$, where T is the thermodynamic temperature and K is the unit symbol for Kelvin, and has scale marks at 0, 1, 2, 3, 4, and 5. If the ordinate value of a point on a curve in the graph is estimated to be 3.2, the corresponding temperature is $T/(10^3 \text{ K}) = 3.2$ or $T = 3200 \text{ K}$. Notice the lack of ambiguity in this form of labeling compared with the technically incorrect way as 'Temperature (10^3 K).' However, an expression such as $\ln(p/\text{MPa})$, where p is the quantity symbol for pressure and MPa is the unit symbol for Megapascal, is perfectly acceptable, because p/MPa is the numerical value of p , when p is expressed in the unit MPa and is simply a number.

Space between Numerical value and Unit Symbol

When it is necessary to express a quantity, the unit symbol should be placed after the numerical value and a space should be kept between the numerical values and the unit symbol. For examples ' 1 m ' or ' 50 kg ' are correct, but ' 1m ' or ' 50kg ' are not. Temperature in Celsius is written as $t=30.2 \text{ }^\circ\text{C}$, not $t=30.2^\circ\text{C}$ or $t=30.2^\circ \text{ C}$. The numerical values having more than four digits on either side of the decimal mark are separated into groups of 'three' using a fixed space counting from both the left and right of the decimal marks, and commas should not be used to separate digits into groups of three. For example the correct way is ' $15\ 739.012\ 53$ ' and not ' 15739.01253 ' or ' $15,739.012\ 53$ '. However, it should be noted that, this rule does not apply for the unit symbols of, angle, minute, and second for a plane angle: $^\circ$, $'$, and $''$. In this case, no space should be left between the numerical value and the unit symbol. For example, a plane angle should be expressed as $\alpha = 60^\circ 22' 8''$ but not as $\alpha = 60^\circ 22' 8''$. However, when unit names are spelt out, the normal rules in English should be followed. For example 'a roll of 35-millimeter film' is acceptable.

Number of Units per Value of a Quantity

The value of a quantity should be expressed with no more than one unit. For example 'Length, $L = 10.234 \text{ m}$ ' is correct, but not ' $L = 10 \text{ m } 23 \text{ cm } 4 \text{ mm}$ '. However, again expressing the values of time intervals and of plane angles are exceptions to this rule. It is possible to express the degree decimally as 22.20° rather than as $22^\circ 12'$.

Unacceptability of attaching Information to Units

When expressing the value of a quantity, it is incorrect to attach letters or other symbols to the unit. Instead, the letters or other symbols should be attached to the quantity. For example: $V_{\max} = 1000 \text{ V}$ is correct, but $V = 1000 V_{\max}$ is not.

Unacceptability of mixing Information with Units

Any information concerning the quantity or its conditions of measurement must be presented in such a way as not to be associated with the unit. For example, 'the neutron emission rate of $5 \times 10^{10}/\text{s}$ ' is correct but 'the emission rate of $5 \times 10^{10} \text{ n/s}$ ' is not. Similarly, 'the number density of oxygen atoms is $3 \times 10^{18}/\text{cm}^3$ ' is correct but 'the density is 3×10^{18} oxygen atoms/ cm^3 ' is not. Another example is 'the resistance per square is 100Ω ' is correct but 'the resistance is $100 \Omega/\text{square}$ ' is not.

Clarity in Writing values of Physical Quantities

Since the value of a physical quantity is expressed as the product of a number and a unit, the position of the unit symbol should be written in such a way as to show which unit symbol belongs to which numerical value of the quantity. For example '51 mm x 50 mm x 2 mm' is correct but '51x50x2 mm' is not. Further, whenever it is necessary to express a range of values for a quantity, it is recommended to use the word 'to' instead of using 'range dash' (-). Some examples are given below:

'225 nm to 2400 nm' is correct, but '225 to 2400 nm' is not.

'0 V to 5 V' is correct but '0-5 V' is not.

'63.2 m \pm 0.1 m' or '(63.2 \pm 0.1) m' is correct but '63.2 \pm 0.1 m' is not.

Choosing SI prefixes

It is often recommended that, for ease of understanding, prefix symbols should be chosen in such a way that numerical values are between 0.1 and 1000, and that only prefix symbols that represent the number 10 raised to a power that is a multiple of 3 should be used.

For example:

$3.3 \times 10^7 \text{ Hz}$ may be written as $33 \times 10^6 \text{ Hz} = 33 \text{ MHz}$

2703 W may be written as $2.703 \times 10^3 \text{ W} = 2.703 \text{ kW}$ or

$5.8 \times 10^{-8} \text{ m}$ may be written as $58 \times 10^{-9} \text{ m} = 58 \text{ nm}$

ppm, ppb, and ppt

The language-dependent terms such as 'part per million', 'part per billion', and 'part per trillion', and their respective abbreviations 'ppm,' 'ppb,' and 'ppt' are not acceptable for use with SI units to express the values of quantities. Forms such as those given in the following examples should be used instead.

'a shift of 1.1 nm/m' but not as 'a shift of 1.1 ppb'

'a sensitivity of 2 ng/kg' but not as 'a sensitivity of 2 ppt'

Significant Figures and Decimal places

When a number is used to describe a physical quantity, each of its digits should carry some meaning, in other words, each should be a significant figure. For example, stating that a certain length L is 6.00 mm should mean that L really is 6.00 mm, rather than 6.01 mm or 5.99 mm. The use of three significant figures in this case implies that the length has been determined to the nearest 0.01 mm and is therefore known to be somewhere between 5.995 000 00 mm and 6.004 999 99 mm. It would be quite wrong, for example, having determined a length as 6.00 mm to the nearest 0.01 mm to write that length as 6.000 mm. The presence of the last zero would imply that the length was in the range 5.999 500 00 mm to 6.000 499 99 mm, which would not be justified. Thus, the number of significant figures in a value should indicate the *precision* with which that value is known.

References and Further Reading

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