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An optimal lockdown relaxation strategy for minimizing the economic effects of covid-19 outbreak in Sri Lanka

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Abstract

In order to recover the damage to the economy by the ongoing covid-19 pandemic, Sri Lanka is undergoing a gradual transition from strict lockdowns to partial lockdowns through relaxation of curfew and other moderated preventive measures. In this work, we propose an optimal lockdown relaxation strategy, which is aimed at minimizing the damage to the economy, while confining the covid-19 incidence to a level endurable by the available healthcare capacity in the country. The relevant optimization model turns out to be non-linear. We use the technique of separable programming to generate solutions and discuss the results.

Keywords: covid-19, separable programming, operational research, health planning

1. Introduction

The ongoing pandemic of the covid-19, which has recently been declared by the World Health Organization as the era-defining global health crisis (Meade, 2020), is an unprecedented threat to the world. It has caused numerous deaths and health complications, upended the day-to-day life and destabilized economies. Sri Lanka reported the first covid-19 case in a Chinese tourist on 27th January 2020 and subsequently in a local person on 11th March, 2020. Aimed at controlling the pandemic, the government of Sri Lanka implemented several strategies, of which reducing human mobility was the most prominent one (Erandi, 2020). A strict strategy of lockdown was enforced together with other preventive measures including case detection, identification of contacts, quarantine, travel restrictions and isolation of small villages as well. Despite those preventive measures, the pandemic continues to threaten the public with daily reported cases. As seen from Figure 1, the reported covid-19 cases in Sri Lanka is still on the rise thus the country is still at risk. Further, Sri Lanka has experienced fluctuating doubling times below 70, as depicted in Figure 2, which questions the appropriateness of relaxations.

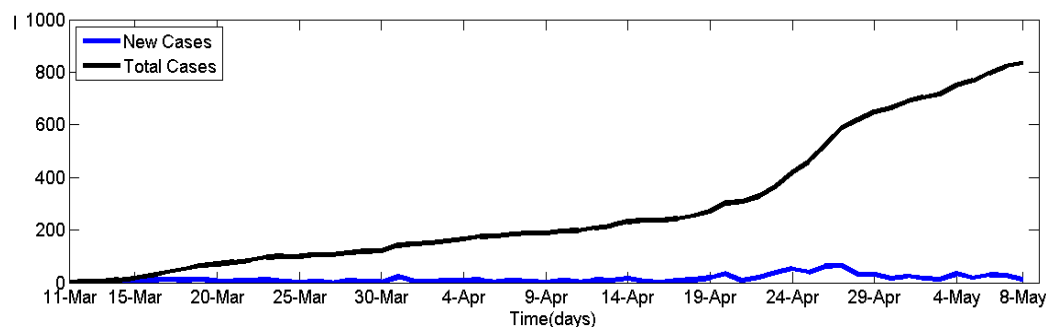


Fig.1. Reported covid-19 cases from 11th March to 8th May, Sri Lanka (Source: Epidemiology Unit, Ministry of Health, Sri Lanka)

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On the other hand, the impact of the preventive measures to the economy of Sri Lanka was significant. Sri Lankan economy was slowly recovering from the Easter Sunday attacks in April 2019, and the Central Bank of Sri Lanka (CBSL) was expecting an economic growth of 4.5-5% together with the political stability after the recently held presidential elections. In this context, a continuing lockdown was regarded as quite impossible for the small island nation. Therefore, the government of Sri Lanka declared that the activities of the country will be restarted from 11th May onwards, subject to several restrictions, aimed at a resuming to ordinary life gradually. The restrictions include social distancing, limitation of the workforce at workplaces etc. Also the nationwide curfew imposed for 52 days was lifted for many regions, except for several regions identified as high-risk zones. A limited number of citizens in high-risk zones were allowed to travel, based on the last digit of their national identity card numbers. Accordingly, specific days are prescribed for the individuals who possess national identity card numbers ending up at different digits, intending to reduce the human movement inside those zones by more than 80%. In addition, one third of the workforce in state institutions in several areas were required to report to work. Also the public transport services were made allowed with strict restrictions on the numbers of passengers. Thus, the country is undergoing a transition from strict lockdowns towards partial lockdowns.

It must be noted that even when the strict lockdowns were implemented in the country's most populous Western province and the high-risk North Western province, partial lockdowns were implemented in several other provinces. For instance, human mobility in the North Central province which contributes most to agriculture was least restricted, a decision which helped the minimally interrupted distribution of rice and vegetables to the regions on which strict curfew was imposed. Also, the manufacturing industries were permitted to operate to a certain extent, prioritizing food and beverages. This was done by allowing a certain percentage of workers at food manufacturing industry to go to work in some regions, subject to strict measures on distancing. Moreover, temporary relaxations of curfew were exercised in several regions at different occasions. Therefore, the country has already undergone partial lockdowns, where different regions were operated to different extents, eventually contributing to the sustainment of the nation's economy during the lockdowns.

Nevertheless, the 52 days of lockdown should not be seen as identical to the new post-lockdown period. A major distinction of the new period from the lockdown days is the *inter-regional travel* which was allowed from 11th May onwards. Recall the major control measure for covid-19 was strictly restricting the human movement inside the country, its relaxation must have unforeseen consequences. Several previous works on epidemiology have pointed out the significance of human mobility to the transmission of covid-19 (Shim et al., 2020, Gong et al., 2020). Therefore, the relaxation of travel restrictions must put a different complexion on the matter, and make the post-lockdown period significantly different to the 52 days' lockdown period.

In this context, it is important to seek what preventive measures are most appropriate to keep the pandemic controlled, while causing the minimum damage to the economy of the country. In other words, it is important to design successful partial lockdown strategies, or, lockdown relaxation strategies, by taking into account the transmission of the disease when travel is permitted and also the economic concerns.

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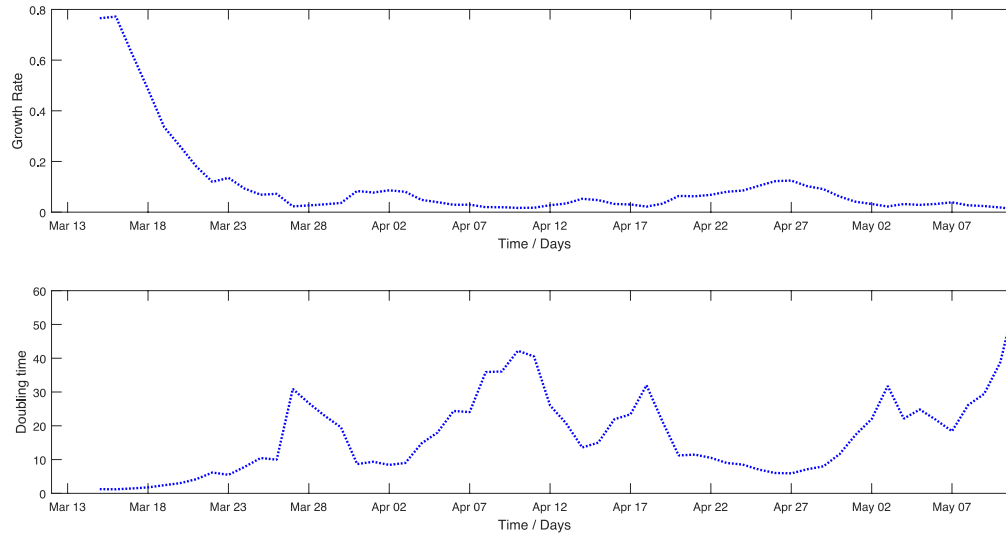


Fig.2. Growth rate and doubling time for covid-19 cases from 14th March to 11th May, Sri Lanka (Source: Epidemiology Unit, Ministry of Health, Sri Lanka)

We propose a region-based lockdown relaxation strategy, which shares certain similarities with what was implemented during the lockdown period. Accordingly, we propose to determine the extent of lockdown a region must undergo, in order to limit the number of covid-19 patients to a number that can be provided necessary healthcare facilities using the current capacity in the country. It is a well known fact that the death rate from covid-19 rapidly increases when a country is unable to provide intensive care unit (ICU) beds and other necessary health facilities to its patients. Therefore, we assume that the country can reinforce a certain number of patients, during a given time. Also we consider the transmission dynamics of covid-19 from available data. Thus, we build an optimization model which determines the extent of lockdown that must be imposed on a region during the post-lockdown period.

The remainder of the paper is organized as follows. In section 2 we forecast the post-lockdown transmission of covid-19 using compartment models in epidemiology, by incorporating the inter-regional mobility factor. In section 3 we develop the optimization model. Section 4 includes our computational techniques and section 5 the interpretation and the discussion of the results we obtained. We conclude the paper in section 6.

2. Post-lockdown disease transmission

In order to model the disease transmission in the country, we adopt the SIR (susceptible-infectious-recovered) model for epidemic transmission, which is the most frequently used compartment model to forecast an epidemic. This model captures the transmission dynamics of diseases of which the infection confers permanent immunity. The population (N) is divided into the three disjoint classes, namely, *susceptible* (S), *infectious* (I) and *recovered* (R). Once the susceptible individuals become infected with the disease, they move to the infectious class. The infected persons move to the recovered class when they get recovered from the disease and a person in the recovered class is assumed to have permanent immunity. Let β denote the

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transmission rate from infected individuals to susceptible individuals and γ denote the recovery rate of infected individuals. Then, the timely variation of the compartments is described by the following set of differential equations:

$$\frac{dS}{dt} = -\beta \frac{S}{N} I \quad (1a)$$

$$\frac{dI}{dt} = \beta \frac{S}{N} I - \gamma I \quad (1b)$$

$$\frac{dR}{dt} = \gamma I \quad (1c)$$

Several researchers have used the SIR model to forecast the covid-19 incidence (Atkeson, 2020; Bastos & Cajueiro, 2020; Roda et al., 2020). The SIR model in its original form is however not very helpful for our purpose. Recall inter-provincial travel is the main characteristic which distinguishes the post-lockdown period in our context, in order to forecast the transmission inside Sri Lanka during that period, it is essential to incorporate the travel component to the conventional SIR model. Hence, we modify the SIR as follows.

Let w_{ij} denote the percentage of daily travels from i th province to the j th province and degree of social distancing respectively. Let x_i denote the lockdown relaxation percentage of the i th province. Then the susceptible (S_i), infectious (I_i), recovered (R_i) populations in i th province can be described by the following set of modified differential equations.

$$\frac{dS_i}{dt} = -\beta \frac{S_i}{N_i} (x_i I_i + \sum_{j \neq i} x_j w_{ij} I_j) \quad (2a)$$

$$\frac{dI_i}{dt} = \beta \frac{S_i}{N_i} (I_i + \sum_{j \neq i} w_{ij} I_j) - \gamma I_i \quad (2b)$$

$$\frac{dR_i}{dt} = \gamma I_i \quad (2c)$$

Following the works by (Miller, 2012; 2017), we define the expected number of transmissions an individual has received by time t as:

$$\xi(t) = \beta \int_0^t \frac{S_i}{N_i} (x_i I_i + \sum_{j \neq i} x_j w_{ij} I_j) dt^* \quad (3)$$

Notice that,

$$\frac{d \left(S_i \exp \left(\int_0^t \beta \frac{S_i}{N_i} (x_i I_i + \sum_{j \neq i} x_j w_{ij} I_j) dt^* \right) \right)}{dt} = \exp \left(\int_0^t \beta \frac{S_i}{N_i} (x_i I_i + \sum_{j \neq i} x_j w_{ij} I_j) dt^* \right) \left(\frac{dS_i}{dt} + \beta \frac{S_i}{N_i} (x_i I_i + \sum_{j \neq i} x_j w_{ij} I_j) \right) = 0 \quad (4)$$

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Thus, it follows that,

$$S_i(t) = S_{i0} \exp(-\xi(t)) \quad (5)$$

Further, by integration Equation 3, it is possible to show that

$$R_i(t) = R_{i0} + \int_0^t \gamma I_i dt^* \quad (6)$$

By substituting $S_i(t)$ and $R_i(t)$ into $N_i(t) = S_i(t) + I_i(t) + R_i(t)$, infected human population at time t according to our model can be expressed as:

$$I_i(t) = N_i(t) - S_{i0} \exp(-\xi(t)) - \left(R_{i0} + \int_0^t \gamma I_i dt^* \right) \quad (7)$$

3. Proposed optimization model

Several works have investigated the critical subject of intensive care management during covid-19 pandemic with different opinions (Emmanuel et al., 2020; Phua et al., 2019; Truog et al., 2020), which we do not wish to treat separately. Determining who would be provided with ICU facilities is another decision problem we do wish to address in this work. Instead, we consider a situation where all patients are provided ICU beds. If the epidemiological recommendations suggest otherwise, this assumption can be readily relaxed and the relevant term can be replaced by the percentage of the infected population who are facilitated with ICU beds. Accordingly, we state the condition that the number of covid-19 patients in the post-lockdown era, which is given by Equation 7 must not exceed the number of ICU beds.

Once the transmission of the disease to different provinces during the post-lockdown period is formulated, it is now important to examine the contribution of these provinces to the economy. Recall the decisions on curfew were made at different notes on different regions during the 52 days of lockdown, a primary intention was sustaining the nationwide economy by considering the economic contribution from regions. Our model is also based on the regular (or the pre-lockdown) economic contribution by different provinces to the economy of Sri Lanka, which is given in Table 1.

Table 1

Agriculture and industrial contribution from each province to the economy of Sri Lanka (Source: Economic Statistics of Sri Lanka, Department of Census and Statistics, Sri Lanka)

Province	Agricultural contribution (%)	Industrial contribution (%)
Western	2.03281	13.79459
Central	12.05421	11.08724
Southern	10.41369	11.73184
Northern	10.23538	10.39966
Eastern	9.700428	10.78642
North Western	9.379458	14.00945
North Central	15.29957	8.293941
Uva	18.61626	6.574989
Sabaragamuwa	12.26819	13.32187

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Since our intention is determining to what extent a province would operate during the post-lockdown era, the decision variable in our optimization model must be the extent of lockdown relaxation of the i th province, given as a percentage, symbolized by x_i in the previous section. Then the inputs follow as given below.

p_i	-	Economic productivity of the i th province
w_{ji}	-	Human mobility between the i th and the j th provinces
β	-	Transmission rate of covid-19
γ	-	Recovery rate of covid-19
N_i	-	Population of the i th province
M	-	Number of ICU beds available in the country
S_{i0}	-	Initial susceptible population in the i th province
R_{i0}	-	Initial recovered population in the i th province

Now, the relevant optimization problem can be expressed as follows.

Maximize

$$Z = \sum_{i=1}^9 p_i x_i \quad (8)$$

subject to

$$\sum_{i=1}^9 I_i \leq M \quad (9a)$$

$$\forall i \in \{1, \dots, 9\}, \ln \left| \frac{I_i - N_i + R_{i0} + \gamma \int_0^t I_i(t^*) dt^*}{-S_{i0}} \right| = -\beta \left(x_i \int_0^t I_i(t^*) dt^* + \sum_{j \neq i} x_j w_{ij} \int_0^t I_j(t^*) dt^* \right) \quad (9b)$$

$$\forall i \in \{1, \dots, 9\}, 0 \leq x_i \leq 1 \quad (9c)$$

The objective function 8 maximizes the total contribution to the economy by all provinces. Constraint 9a assures that the number of total infected persons within the relevant period of time does not exceed the number of ICU beds in the country. The inter-regional transmission of the disease as obtained by applying the SIR model is given by the set of constraints in 9b. Finally, constraint in 9c assures that the percentage of lockdown of any province must be between 0% and 100%.

4. Solution technique

In order to solve the non-linear optimization problem given by Equations 8 and 9, we consider a particular characteristic of the objective function 8 and constraints 9a, 9c. That is, despite being multivariate functions, all these are expressible as sums of single-variable functions. Also the constraint 9b is readily transformable to this form by substituting $\ln \left| \frac{I_i - N_i + R_{i0} + \gamma \int_0^t I_i(t^*) dt^*}{-S_{i0}} \right| = h_i$ and $f_i = \exp(-h_i)$. Therefore, we can restate our optimization problem as follows.

Maximize

$$Z = \sum_{i=1}^9 p_i x_i \quad (10)$$

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subject to

$$\sum_{i=1}^9 I_i \leq M \quad (11a)$$

$$\forall i \in \{1, \dots, 9\}, h_i = -\beta \left(x_i \int_0^t I_i(t^*) dt^* + \sum_{j \neq i} x_j w_{ij} \int_0^t I_i(t^*) dt^* \right) \quad (11b)$$

$$\forall i \in \{1, \dots, 9\}, I_i = -S_{i0} f_i + N_i - R_{i0} - \gamma \int_0^t I_i(t^*) dt^* \quad (11c)$$

$$\forall i \in \{1, \dots, 9\}, f_i = \exp(-h_i) \quad (11d)$$

$$\forall i \in \{1, \dots, 9\}, 0 \leq x_i \leq 1 \quad (11e)$$

This reformulation given by Equations 10 and 11 motivates us to adopt the technique of *separable programming*. This technique was first introduced in (Charnes & Lemke, 1954) for constrained optimization of non-linear convex functions, whenever these functions are *separable*; that is, expressible as sums of functions of single variable. Since its inception, separable programming has been a very useful optimization technique, with applications to several real-world problems including agricultural planning (Thomas et al., 1972), linear complementarity problem (Bard & Falk, 1982), Newsboy problem (Abdel-Malek & Otegbeye, 2013) and demand allocation (Hassan & Abdelghany, 2017). The main tool in separable programming is, replacing the non-linear functions in the optimization problem by piecewise linear approximations.

Notice that Equation 11d in our formulation is non-linear and hence piecewise linearization is required for the function f_i . We divide the domain of f_i , that is $[0, D_i]$, into n subdivisions, each of length d by defining h_{ik} as follows, where, D_i is the maximum value of h_i .

$$0 = h_{i1} \leq h_{i2} \leq \dots \leq h_{in} = D_i \quad (12)$$

where

$$|h_{ik} - h_{i(k+1)}| = d, \forall k \in \{1, \dots, n-1\} \quad (13)$$

Then any point h_i in the interval $[0, D_i]$ can be uniquely expressed as:

$$h_i = \lambda_{ik} h_{ik} + \lambda_{i(k+1)} h_{i(k+1)} \quad (14)$$

where

$$\lambda_{ik} + \lambda_{i(k+1)} = 1 \text{ and } \lambda_{ik}, \lambda_{i(k+1)} \geq 0 \quad (15)$$

Now the piecewise linear approximation to f_i is expressible as:

$$f_i^* = \sum_{k=1}^n \lambda_{ik} f_i(h_{ik}) \quad (16)$$

where

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$$h_i = \sum_{k=1}^n \lambda_{ik} h_{ik} \quad (17a)$$

$$\sum_{k=1}^n \lambda_{ik} = 1 \quad (17b)$$

$$\forall k \in \{1, \dots, n\}, \lambda_{ik} \geq 0 \quad (17c)$$

with the additional restriction at most two adjacent λ_{ik} 's are positive.

Replacing the non-linear functions f_i in Equation 11d by linear approximations in Equation 17, our problem can be restated again as follows:

Maximize

$$Z = \sum_{i=1}^9 p_i x_i \quad (18)$$

subject to

$$\sum_{i=1}^9 I_i \leq M \quad (19a)$$

$$\forall i \in \{1, \dots, 9\}, \sum_{k=1}^n \lambda_{ik} h_{ik} = -\beta \left(x_i \int_0^t I_i(t^*) dt^* + \sum_{j \neq i} x_j w_{ij} \int_0^t I_i(t^*) dt^* \right) \quad (19b)$$

$$\forall i \in \{1, \dots, 9\}, I_i = -S_{i0} f_i + N_i - R_{i0} - \gamma \int_0^t I_i(t^*) dt^* \quad (19c)$$

$$\forall i \in \{1, \dots, 9\}, f_i = \sum_{k=1}^n \lambda_{ik} f_i(h_{ik}) \quad (19d)$$

$$\sum_{k=1}^n \lambda_{ik} = 1 \quad (19e)$$

$$\forall k \in \{1, \dots, n\}, \lambda_{ik} \geq 0 \quad (19f)$$

$$\forall i \in \{1, \dots, 9\}, 0 \leq x_i \leq 1 \quad (19g)$$

with the additional restriction that at most two adjacent λ_{ik} 's are positive.

Except for the additional restriction on adjacency, the approximated problem given by Equations 18 and 19 is a linear program, readily solvable by the simplex method. It is a standard fact in separable programming that, in case of maximization, if the approximated objective function is

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concave and each piecewise linear constraint is convex, then the solution of the linearly approximated formulation without the additional restriction is feasible to the original problem (Sinha, 2005; Hochbaum & Shanthikumar, 1990). From this, the computational hardness implied by the non-linearity could be readily overcome, and the problem becomes efficiently solvable.

5. Results

Based on data and information available on covid-19 pandemic in Sri Lanka, we made substitutions of numerical values to the inputs. First, Sri Lanka currently has approximately 500 ICU beds, and the authorities have recently declared their willingness to increase this to 1000. Therefore, the input M was set to 1000 in our primary computational experiment. Further, the covid-19 incidence by province as per 11th May were taken as the initial conditions. This selection of initial condition is a realistic choice required by the model, as it is the day the country started undergoing relaxations. Provincial economic contribution was substituted as in Table 1. The duration the model applied was taken as one week from the day the relaxations started.

The global optimum to the problem achieved through separable programming, applied to these data, provided the best relaxation percentages of the provinces, as given by Table 2 and visualized in Figure 3. There are several interesting and counterintuitive implications. According to these results, no relaxation must be done on the lockdown of the Western province; moreover, it is not the only province which must be kept at strict lockdowns; also the Central and the North Western provinces must undergo strict lockdowns. On the other hand, total relaxation is possible for five provinces, namely, Southern, Northern, Eastern, North Central and Uva. Recall the North Central province, the country's agricultural hub, underwent least lockdown measures till 11th May to assure the people are provided with rice and vegetables, our results too indicate that the agricultural farming can be restarted in this province and its economic centres could be kept open during the post-lockdown period.

Table 2
Optimal lockdown relaxations of provinces as percentages (Number of ICU beds =1000)

Province	x_i
Western	0
Central	0
Southern	100
Northern	100
Eastern	100
North Western	0
North Central	100
Uva	100
Sabaragamuwa	57

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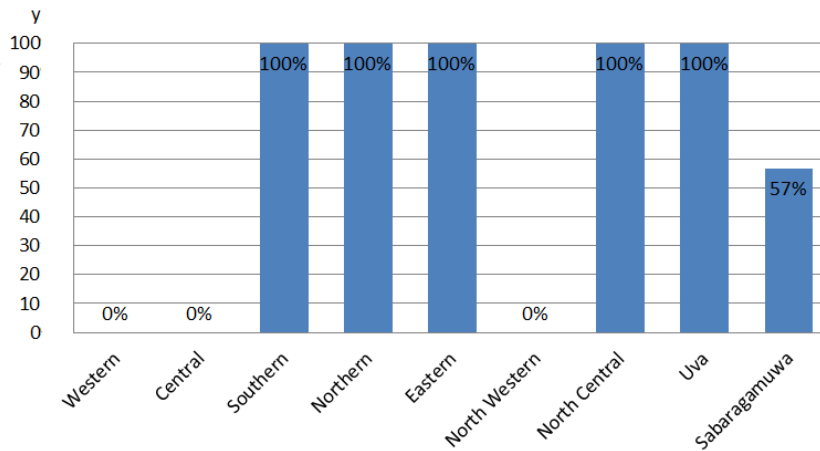


Fig.3. Optimal results for lockdown relaxation (y - Relaxation level as a percentage, Number of ICU beds =1000)

Our secondary computational experiment was on investigating the scenario when different numbers of ICU beds are available. The results are shown in Figure 4. Recall Sri Lanka currently has 500 ICU beds in state hospitals, the results indicate that relaxations must be applied only to three provinces, namely, Southern, Eastern and North Central. The Northern province also could undergo a minor relaxation, if the number is increased to 750, in addition to the significant fact that Uva province might undergo a major relaxation. The lockdown imposed on the Central province can be partly relaxed at a stage when the number of ICU beds is in between 1000 and 1250. Sabaragamuwa province could undergo a partial relaxation when this number is between 750 and 100. Western and North Western provinces must remain unrelaxed even if the number is increased to 2000.

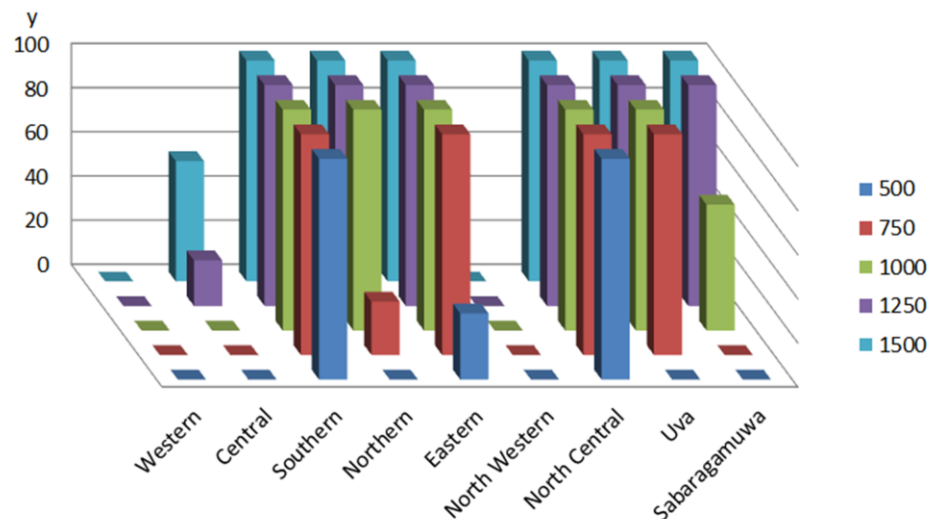


Fig.4. Variation of the optimal lockdown relaxation levels of provinces (as percentages) with the total number of ICU beds available in the country

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6. Conclusion

Human mobility is a critical factor in transmitting communicable diseases. Accordingly, lockdowns with strict travel restrictions were implemented in several countries to fight against the covid-19 pandemic. A nationwide curfew was implemented in Sri Lanka, which had a significant impact on its economy. Since minimizing both covid-19 incidence and economic effects are two conflicting goals, the government of Sri Lanka declared its readiness to resume to activities and decided to partly relax the lockdowns, aimed at a gradual transition to ordinary life. Instead of ad-hoc lockdown or relaxation strategies, in this work we proposed a systematic lockdown relaxation strategy that can help in achieving the two conflicting goals.

Considering the potential disease transmission during the post-lockdown period, as given by the epidemic models, we proposed an optimization model, from which the optimal region-based lockdown strategies were determined, while confining the covid-19 incidence to a number that is endurable to the country and minimizing the damage to its economy. In particular, we proposed to determine the extent of lockdown relaxation for each region such that all covid-19 patients could be provided ICU facilities and the contribution to economy by all provinces is maximized. Since the resulting optimization problem turned out to be non-linear and several of its functions were expressible as sums of single-variable functions, we adopted the method of separable programming to generate solutions. Accordingly, we converted the non-linear functions to piecewise linear approximations and found the global optimum.

In a more realistic setting, other constraints must be added to our model. For instance, it was mentioned in the introduction that during the lockdown period, the North Central province which acts as the agricultural hub in the country underwent least lockdown restrictions. If all agrarian activities must be continued in the post-lockdown period, the relevant constraint can be readily incorporated into our model, as a lower bound to the relaxation of this province. That adds another linear, single-variable and convex constraint which does not change the solution criterion. However, if the government is looking forward to running different specific industries in different regions, our model needs further modifications. In that case, a decision variable could be indexed by two indices, one for the province and the other for the industry. Further, our model can be applied at any moment in the post-lockdown period by substituting relevant initial conditions. In addition, even if travel restrictions were reimposed on certain roads, the relevant input can be changed and the updated optimal lockdown levels could be determined by minimum modifications. Due to concavity and convexity of the objective functions and constraints, it is efficiently solvable, even if the inter-provincial mobility information were replaced by a subtler dataset such as inter-district mobility; which could be a significant improvement of our model in light of accuracy. Accordingly, our optimization model can be improved further to help the decision making process in sustaining the economy in post-lockdown Sri Lanka, preventing the excessive transmission of covid-19. It would be an interesting future research task to formulate an analogous optimization problem, in which the factors that integrate the individual economic contribution by provinces are included.

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