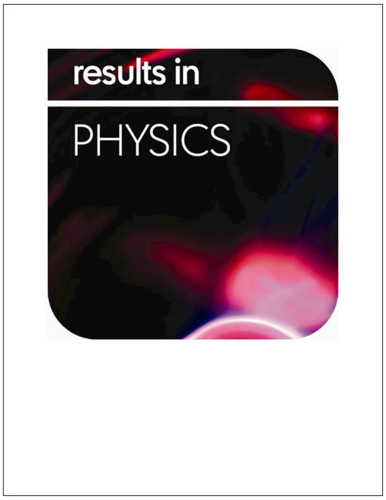

A New Extended Rayleigh Distribution with Applications of COVID-19 Data.

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A New Extended Rayleigh Distribution with Applications of COVID-19 Data .

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Abstract

This paper aims to model the COVID-19 spread in Italy, Mexico and Netherlands, by specifying an optimal statistical model to analyze the mortality rate of COVID-19. A new lifetime distribution with three-parameter is introduced by a combination of Rayleigh distribution and extended odd Weibull family to formulate the extended odd Weibull Rayleigh (EOWR) distribution. This new distribution has many nice properties as simple linear representation, hazard rate function and moment generating function. Maximum likelihood, maximum product spacing and Bayesian estimation methods are applied to estimate the unknown parameters of EOWR distribution. MCMC method is used for the Bayesian estimation. A numerical result of Monte Carlo simulation is obtained to assess the use of estimation methods.

Keywords**:** extended odd Weibull family , Rayleigh distribution; Bayesian, COVID-19; maximum product spacing.

2000 MSC: 60E05, 62F10

1 Introduction

One of the main tasks of statistics is to be modeling the natural life events in the form of probability distributions. Probability distributions are used to modeling natural life phenomena that are characterized by uncertainty and riskiness. Many of the probability distributions have been derived because the natural life phenomena are complex and diversified. However, known probability distributions remain unable to accurately represent data for some natural phenomena. These lead to the expansion and Modification of generalized probability distributions. Generalized probability distributions have been progressing with the popular character of having adding parameters. The addition of some parameters to the known probability distributions improved the quality of suitability for the natural phenomena data and higher accuracy of describing the shape of the tail of the distribution.

Most of extensions of the Rayleigh probability distribution have been derived because of their great importance in describing many natural life phenomena. The Rayleigh probability density function, attributed to Lord Rayleigh (1842-1919), it concerned with describing skewed data Rayleigh [1] Many researchers consider one scale parameter Rayleigh, like Robert C.P. Diebolt and Robert [2] discussed deviation and distance measure in economic, which can be applied in another real phenomena data. The Extended of probability distribution was originally introduced by Lehmann [3]. Kundu and Raqab [4] provided a generalization of the Rayleigh probability distribution and estimated its unknown parameters using several different methods. In Voda [5] used the conservative technique to derive a new generalization of the Rayleigh probability distribution. In Dey [6] presented Bayesian estimates of the Rayleigh probability distribution parameters using the Linux loss functions and error square. In Merovci [7] used the square ordinal transformation method in developing the Transmuted Rayleigh probability distribution. In Merovci and Elbatal [8] presented a Weibel-Rayleigh probability distribution. Based on type II censored data, in Mahmoud and Ghazal [9] discussed the parameters estimation of exponentiated Rayleigh. In Ateeq et al. [10] RayleighRayleigh derived the distribution (RRD) using Transformed Transformer technique. El-Sherpieny and Almetwally [12] introduced Bivariate generalized Rayleigh distribution based on clayton copula with different applications. In Almetwally et al. [11] maximum likelihood and maximum product spacing estimates for generalized Rayleigh distribution based on the adaptive type-II progressive censoring schemes. In Al-Babtain [13] proposed a new extension of the Rayleigh distribution with two parameter called type I half logistic Rayleigh distribution.

Here we study a new model with three parameters, it is called extended odd Weibull Rayleigh (EOWR) distribution. The EOWR distribution is obtained based on the extended odd Weibull-G (EOW-G) family, which introduced by Alizadeh et al. [14]. Let $\overline{G}(x;\theta) = 1 - G(x;\theta)$ and $g\left(x;\theta\right) = \frac{dG(x;\theta)}{dx}$ denote the survival function (S) and probability density function (PDF) of a baseline model with parameter vector θ respectively, so the CDF of the EOW-G family is given by:

$$F(x;\alpha,\beta,\theta) = 1 - \left\{1 + \beta\left[\frac{G(x;\theta)}{\overline{G}(x;\theta)}\right]^{\alpha}\right\}^{\frac{-1}{\beta}}, x \in \mathbb{R}. \tag{1}$$

The corresponding PDF of (1) is defined by

$$f(x;\alpha,\beta,\theta) = \frac{\alpha\, g(x;\theta)\, G(x;\theta)^{\alpha-1}}{\overline{G}(x;\theta)^{\alpha+1}}\left\{1 + \beta\left[\frac{G(x;\theta)}{\overline{G}(x;\theta)}\right]^{\alpha}\right\}^{\frac{-1}{\beta}-1}, x \in \mathbb{R}, \tag{2}$$

where α and β are positive shape parameters. The random variable with PDF (2) is denoted by $X \sim$EOW-G(α, β, θ). Afify and Mohamed [15] introduced a new flexible three-parameter exponential distribution called the extended odd Weibull exponential distribution. Alshenawy et al. [16] maximum likelihood and maximum product spacing estimates of the extended odd Weibull exponential distribution have been discussed under progressive type-II censoring scheme with random removal.

Our goal is to study point estimation of the unknown parameters of EOWR by using four classical methods of estimation and Bayes estimation method. A statistical comparison between these methods is conducted via simulation to asses the performance of these methods and to study how these estimators behave for several sample sizes and parameter values.

The rest of this paper is organized as follows. In Section 2, we define EOWR distribution. EOWR linear representation of its PDF is obtained in Section 3, along with some of its statistical properties. Three methods of point estimation are studied in Section 4. In Section 5, a simulation study is conducted in order to compare the performance of these estimation methods. Three real data sets of COVID-19 from different life applications are used in Section 6 to prove the efficiency of the EOWR distribution with respect to other distributions. Finally, conclusions are given in Section 7.

2 EOWR Distribution

The three-parameter EOWR distribution is a special model of EOW-G family with Rayleigh distribution as a baseline function. The Rayleigh distribution under consideration has PDF and CDF of the form

$g(x;\delta) = 2\delta x e^{-\delta x^2}$ and $G(x;\delta) = 1 - e^{-\delta x^2}$, $x > 0$, $\delta > 0$. By substituting the CDF and PDF of the Rayleigh model in (1) and (2), we obtain the CDF and PDF of the EOWR distribution respectively as;

$$F(x;\alpha,\beta,\delta) = 1 - \left\{1 + \beta\left[e^{\delta x^2} - 1\right]^{\alpha}\right\}^{\frac{-1}{\beta}}, x > 0, \alpha, \beta, \delta > 0. \tag{3}$$

$$f(x;\alpha,\beta,\delta) = 2\alpha\delta x e^{\delta x^2}\left(e^{\delta x^2} - 1\right)^{\alpha-1}\left[1 + \beta\left(e^{\delta x^2} - 1\right)^{\alpha}\right]^{-\frac{1+\beta}{\beta}}, x > 0, \alpha, \beta, \delta > 0. \tag{4}$$

Therefore, a random variable with PDF (4) is denoted by $X \sim$EOWR(α, β, δ). The EOWR model reduces to the two parameter Weibull Rayleigh model when $\beta \to 0^+$.

The hazard rate function (HR) of the EOWR distribution are given by

$$h(x;\alpha,\beta,\delta) = \frac{2\alpha\delta x e^{\delta x^2}\left(e^{\delta x^2} - 1\right)^{\alpha-1}}{1 + \beta\left(e^{\delta x^2} - 1\right)^{\alpha}}$$

Figures 1 and 2 are different shapes of the PDF and HR of the EOWR distribution. These figures show that the PDF of the EOWR distribution can be right-skewed, symmetric or decreasing curves. The HR of the EOWR distribution has some important shapes, including, constant, decreasing, and upside down curve, which are attractive characteristics for any lifetime model. It can be noticed from the application section, that the EOWR distribution possesses great flexibility and can be used to model skewed data, hence widely applied in different areas such as biomedical studies, biology, reliability, physical engineering, and survival analysis.

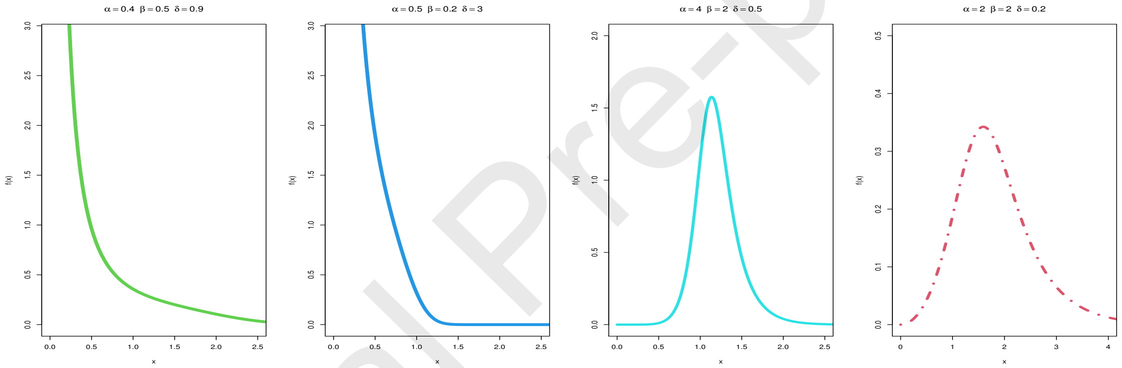

Figure 1: Plots of the probability density function (PDF) of the EOWR distribution.

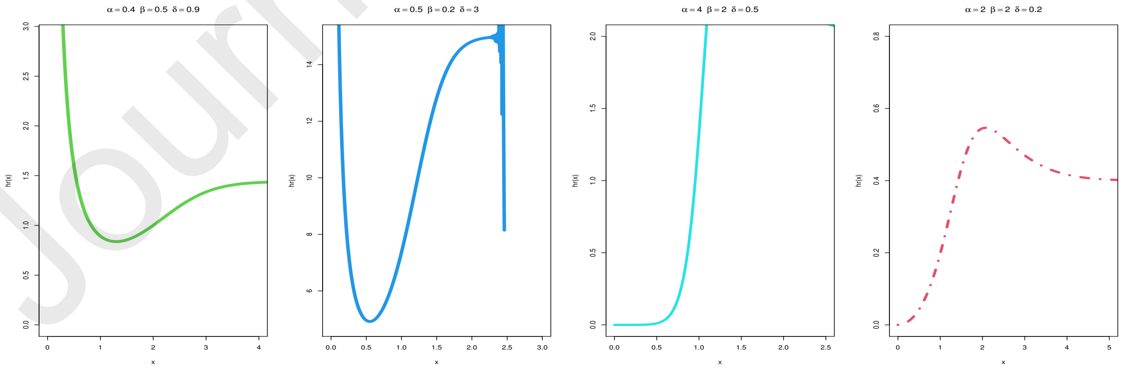

Figure 2: Plots of the hazard rate function (HR) of the EOWR distribution.

3 Statistical Properties

In this section, we observe some statistical properties of the EOWR distribution namely, the linear representation of PDF, which is useful in finding the moments and moment generating function (MGF). Also we obtain the mean residual life and mean inactivity time.

3.1 Linear Representation

Linear representation for the EOWR density using series techniques is useful for finding many statistical values and properties of the needed distribution. Alizadeh et al. [14] showed that EOW-G family has the following mixture representation of its density

$$f(x) = \sum_{j,k=0}^{\infty} a_{j,k}\ h_{\alpha j+k}(x),$$

where $a_{j,k} = \frac{-\beta^j \Gamma(\alpha j+k)(-1/\beta)_j}{k! j! \Gamma(\alpha j)}$ and $h_{\alpha j+k}(x) = (\alpha j+k)\, G(x)^{\alpha j+k-1} g(x)$ is the Exponential-G density with positive power parameter $\alpha j + k$. Now substituting the PDF and CDF of the Pareto distribution, the above equation can be written as

$$f(x) = \sum_{j,k=0}^{\infty} a_{j,k} 2\delta x e^{-\delta x^2} \left(1 - e^{-\delta x^2}\right)^{\alpha j+k-1}. \tag{5}$$

Equation (5) can be written as

$$f(x) = \sum_{j,k=0}^{\infty} \upsilon_{j,k}\ g\left(x; \alpha j + k, \delta\right), \tag{6}$$

where $\upsilon_{j,k} = \frac{a_{j,k}}{(\alpha j+k)}$, and $g\left(x; \alpha j + k, \delta\right)$ denotes Generalized Rayleigh density with $\alpha j + k, \delta$ as parameters of Generalized Rayleigh distribution. Hence the PDF of EOWR can be expressed as a linear combination of Generalized Rayleigh distribution. Let X be a random variable having Generalized Rayleigh distribution. Then, the ith ordinary moment, and MGF of X are

$$\mu_{r,X}(\alpha,\delta) = \alpha\delta(l+1) \sum_{l=0}^{\infty} \binom{\alpha-1}{l} (-1)^l \Gamma\left(\frac{r}{2}+1\right) \left(\delta(l+1)\right)^{\frac{-r-2}{2}}, \tag{7}$$

$$M_X\left(t,\alpha,\delta\right) = \sum_{l=0}^{\infty} \binom{\alpha-1}{l} (-1)^l \left(1 + \sqrt{0.5\delta(l+1)}\, t e^{0.5\delta t^2} \sqrt{0.5\pi}\, erf(0.5\sqrt{\delta} t + 1)\right). \tag{8}$$

3.2 Moments and Moment Generating Functions

The ith moment of X follows directly from Equations (6) and (7)

$$\mu_r = E(X^r) = \sum_{j,k=0}^{\infty} \upsilon_{j,k}\ \mu_{r,X}(\alpha j + k, \delta). \tag{9}$$

Referring to Equation (6), the MGF of the EOWR distribution is given by:

$$M(t) = \sum_{j,k=0}^{\infty} \upsilon_{j,k} M_X\left(t; \alpha j + k, \delta\right).$$

3.3 Quantile Function and Median

The quantile function of EOWR distribution is used in the theoretical aspect of probability theory for this model, statistical applications, and simulations. Simulation algorisms used quantile function to produce simulated random variables. The quantile function (Q) of the EOWR distribution are given by

$$Q(q)=\sqrt{\frac{1}{\delta}\log\left\{1+\left(\frac{1}{\beta}\left[(1-q)^{-\beta}-1\right]\right)^{\frac{1}{\alpha}}\right\}} \tag{10}$$

In particular, the median of EOWR distribution can be derived from 10 be setting q = 0.5. Then, the median is given by $median=\sqrt{\frac{1}{\delta}\log\left\{1+\left(\frac{1}{\beta}\left[2^{\beta}-1\right]\right)^{\frac{1}{\alpha}}\right\}}$.

4 Parameter Estimation

In this section, we use different point estimation methods to estimate the unknown parameters of the EOWR. We use maximum likelihood estimator (MLE), Least Square (LS), maximum product of spacing estimator (MPS) and Bayesian estimation methods. In the last few years, parameter estimation using different estimation methods got great attention by many authors such as Almetwally and Almongy [17], Haj Ahmad and Almetwally [18], Basheer et al. [19] and Afify and Mohamed [15].

4.1 Maximum Likelihood Method

Let $x_1,\cdots,x_n$ be a random sample from the EOWR distribution with parameters α,β, and δ. The likelihood function can be written as:

$$L(\Omega)=2^n\alpha^n\delta^n e^{\delta\sum_{i=1}^{n}x_i^2}\prod_{i=1}^{n}\left\{x_i\left(e^{\delta x_i^2}-1\right)^{\alpha-1}\left[1+\beta\left(e^{\delta x_i^2}-1\right)^{\alpha}\right]^{-\frac{1+\beta}{\beta}}\right\}, \tag{11}$$

the log-likelihood function is

$$\ell(\Omega)\propto\left[\log(\alpha)+\log(\delta)\right]+\delta\sum_{i=1}^{n}x_i^2+(\alpha-1)\sum_{i=1}^{n}\log\left(e^{\delta x_i^2}-1\right)-\left(\frac{1+\beta}{\beta}\right)\sum_{i=1}^{n}\log\left[1+\beta\left(e^{\delta x_i^2}-1\right)^{\alpha}\right], \tag{12}$$

where $H_i=(x_i^{\delta}-1)$ and $\Omega=(\alpha,\ \beta,\ \delta)$ is a vector of the EOWR parameters. The MLE are obtained by solving the following normal equations,

$$\frac{\partial\ell(\Omega)}{\partial\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\log\left(e^{\delta x_i^2}-1\right)-(\beta+1)\sum_{i=1}^{n}\frac{\left(e^{\delta x_i^2}-1\right)^{\alpha}\log\left(e^{\delta x_i^2}-1\right)}{1+\beta\left(e^{\delta x_i^2}-1\right)^{\alpha}}=0,$$

$$\frac{\partial\ell(\Omega)}{\partial\delta}=\frac{n}{\delta}+\sum_{i=1}^{n}x_i^2+(\alpha-1)\sum_{i=1}^{n}\frac{e^{\delta x_i^2}x_i^2}{e^{\delta x_i^2}-1}-\alpha(\beta+1)\sum_{i=1}^{n}\frac{\left(e^{\delta x_i^2}-1\right)^{\alpha-1}x_i^2e^{\delta x_i^2}}{1+\beta\left(e^{\delta x_i^2}-1\right)^{\alpha}}=0,$$

and

$$\frac{\partial\ell(\Omega)}{\partial\beta}=\frac{1}{\beta^2}\sum_{i=1}^{n}\log\left[1+\beta\left(e^{\delta x_i^2}-1\right)^{\alpha}\right]-\frac{\beta+1}{\beta}\sum_{i=1}^{n}\frac{\left(e^{\delta x_i^2}-1\right)^{\alpha}}{1+\beta\left(e^{\delta x_i^2}-1\right)^{\alpha}}=0$$

These equations cannot be solved explicitly, hence a nonlinear optimization algorithm as Newton Raphson method is used.

4.2 Maximum Product Spacing

According to Cheng and Amin [20], the maximum product spacing method (MPS) is an efficient estimation method that proved to have some advantages with respect to other point estimation methods. So we use MPS in this section to have point estimation of the unknown parameters of EOWR distribution. This can be obtained by solving the normal equations resulted from taking partial derivatives of logarithm of product spacing function $G(\mathbf{\Theta})$ which is written as:

$$G(\mathbf{\Omega}) = \left\{ \left(1 - \{1 + \beta\,\Delta(x_1)^{\alpha}\}^{\frac{-1}{\beta}}\right) \left(1 - \{1 + \beta\,\Delta(x_n)^{\alpha}\}^{\frac{-1}{\beta}}\right) \prod_{i=2}^{n} \left[\{1 + \beta\,\Delta(x_{i-1})^{\alpha}\}^{\frac{-1}{\beta}} + \{1 + \beta\,\Delta(x_i)^{\alpha}\}^{\frac{-1}{\beta}} \right] \right\}^{\frac{1}{n+1}}.$$

where $\Delta(x_i) = \left(e^{\delta x_i^2} - 1\right)$ and the logarithmic function of $G(\mathbf{\Theta})$

$$\begin{aligned} \log G(\mathbf{\Omega}) \propto \log\left[1 - \{1 + \beta\,\Delta(x_1)^{\alpha}\}^{\frac{-1}{\beta}}\right] + \log\left[1 - \{1 + \beta\,\Delta(x_n)^{\alpha}\}^{\frac{-1}{\beta}}\right] + \\ \sum_{i=2}^{n} \log\left[\{1 + \beta\,\Delta(x_{i-1})^{\alpha}\}^{\frac{-1}{\beta}} + \{1 + \beta\,\Delta(x_i)^{\alpha}\}^{\frac{-1}{\beta}}\right] \end{aligned} \tag{13}$$

The MPS estimators of $\mathbf{\Omega}$ are obtained by differentiating the log-product equation (13) with respect to each parameter separately, then we solve the nonlinear system of equations found by using any iterative procedure techniques such as Newton Raphson algorithms. This developed in last few year to estimation parameter of model under censoring scheme as Ng et al. [21], Basu et al. [22], Almetwally and Almongy [23], Almetwally et al. [17, 24], and El-Sherpieny et al. [25].

4.3 Bayesian estimation

Bayesian methods is a statistical inference that depends on the choice of the prior distribution and the loss function. In this method all parameters are considered as random variables with certain distribution called prior distribution. If prior information is not available which is usually the case, we need to select one. Since the selection of prior distribution plays an important role in estimation of the parameters, our choice for the priors are the independent gamma distributions. On the other hand, the loss function is important in Bayesian methods. Most of the Bayesian inference procedures are developed under the symmetric and asymmetric loss functions. One of the most common symmetric loss function is the squared error loss function. The independent joint prior density function of Ω can be written as follows:

$$\pi(\Omega) = \frac{h_1^{s_1}}{\Gamma(s_1)} \frac{h_2^{s_2}}{\Gamma(s_2)} \frac{h_3^{s_3}}{\Gamma(s_3)} \alpha^{s_1-1} \beta^{s_2-1} \delta^{s_3-1} e^{-(h_1\alpha + h_2\beta + h_3\delta)}. \tag{14}$$

The joint posterior density function of Θ is obtained from (11) and (14)

$$\pi(\Theta|\underline{x}) = \frac{\ell(\underline{x}|\Theta).\pi(\Theta)}{\int_{\Theta} \ell(\underline{x}|\Theta).\pi(\Theta) d\Theta}. \tag{15}$$

The Bayes estimators of Θ, say $(\widehat{\alpha}_B, \widehat{\beta}_B, \widehat{\delta}_B)$ based on squared error loss function is given by

$$\begin{aligned} \hat{p}_{B-SEL}(\alpha, \beta, \delta) &= E_{(\alpha,\beta,\delta|\underline{x})}[p(\alpha, \beta, \delta)] \\ &= \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} p(\alpha, \beta, \delta) \times \pi(\Theta|\underline{x}) d\alpha d\beta d\delta. \end{aligned} \tag{16}$$

It is noticed that the integrals given by (16) can't be obtained explicitly. Because of that we use the Markov Chain Monte Carlo technique (MCMC) to find an approximate value of integrals in (16). Many of studies used MCMC technique such as, Almetwally et al. [17, 24].

5 Simulation Analysis

In this section Monte-Carlo simulation procedure is performed for comparison between the classical estimation methods: MLE, MPS and Bayesian estimation method under square error loss function based on MCMC, for estimating parameters of EOWR distribution in life time by R language. Monte-Carlo experiments are carried out based on data- generated 10000 random samples from EOWR distribution, where x has EOWR life time for different actual values of parameters and different sample sizes n:(50, 100 and 200). We could define the best estimators methods as which minimizes the bias and root mean squared error (RMSE) of estimators.

Tables 1, 2 summarizes the simulation results of point estimation methods proposed in this paper. We consider the bias and the RMSE values in order to perform the needed comparison between different point estimation methods. The following remarks can be noted from these tables:

Table 1: Bias and RMSE of EOWR distribution for MLE, MPS and Bayesian when $\alpha = 0.5$

$\alpha = 0.5$				MLE		MPS		Bayesian	
β	δ	n		Bias	RMSE	Bias	RMSE	Bias	RMSE
0.5	0.5	50	α	0.0183	0.1240	-0.0059	0.0087	0.0062	0.0414
			β	0.0105	0.8714	0.1152	0.3386	0.1075	0.2227
			δ	0.0269	0.3340	0.0587	0.0665	0.0907	0.1791
		100	α	0.0025	0.0683	-0.0062	0.0040	0.0048	0.0371
			β	-0.0189	0.6002	0.0741	0.1938	0.1272	0.2294
			δ	0.0177	0.2677	0.0422	0.0348	0.0732	0.1392
		200	α	0.0042	0.0460	-0.0030	0.0020	0.0043	0.0318
			β	0.0413	0.3988	0.0685	0.1171	0.1079	0.2137
			δ	0.0289	0.1856	0.0342	0.0203	0.0608	0.1189
	2	50	α	0.0095	0.1031	-0.0155	0.0078	0.0078	0.0418
			β	-0.0498	0.5912	-0.0046	0.1853	0.0772	0.1711
			δ	-0.0202	0.7408	0.0011	0.3149	0.1607	0.3494
		100	α	0.0020	0.0632	-0.0057	0.0035	0.0078	0.0373
			β	-0.0483	0.5419	0.0356	0.1069	0.0751	0.1651
			δ	0.0008	0.9164	0.0654	0.2222	0.1581	0.3380
		200	α	0.0017	0.0467	-0.0028	0.0019	0.0050	0.0325
			β	0.0017	0.3634	0.0485	0.0559	0.0745	0.1608
			δ	0.0411	0.6169	0.0772	0.1242	0.1478	0.3097
2	0.5	50	α	0.0179	0.0883	-0.0225	0.0065	0.0034	0.0402
			β	0.1007	0.6120	-0.0195	0.1984	0.0508	0.0853
			δ	0.0381	0.1775	0.0150	0.0288	0.0561	0.1332
		100	α	0.0137	0.0667	-0.0102	0.0035	0.0045	0.0384
			β	0.2856	1.3758	-0.0040	0.0876	0.0482	0.0834
			δ	0.0718	0.3392	0.0045	0.0112	0.0342	0.0912
		200	α	0.0034	0.0425	-0.0093	0.0017	0.0013	0.0320
			β	0.0785	0.5627	-0.0209	0.0384	0.0454	0.0819
			δ	0.0217	0.1403	-0.0016	0.0052	0.0251	0.0706
	2	50	α	0.0176	0.1017	-0.0213	0.0073	0.0062	0.0382
			β	0.1072	1.0997	-0.0480	0.1761	0.0400	0.0757
			δ	0.0687	0.7861	-0.0452	0.1742	0.0940	0.3061
		100	α	0.0109	0.0658	-0.0111	0.0038	0.0053	0.0376
			β	0.0967	0.8821	-0.0236	0.0966	0.0394	0.0748
			δ	0.1147	0.9710	-0.0105	0.0960	0.0968	0.2719
		200	α	0.0025	0.0470	-0.0094	0.0018	0.0017	0.0307
			β	0.0254	0.5610	-0.0232	0.0602	0.0365	0.0719
			δ	0.0346	0.4849	-0.0115	0.0543	0.0841	0.2240

Table 2: Bias and RMSE of EOWR distribution for MLE, MPS and Bayesian when $\alpha = 2$

$\alpha = 2$				MLE		MPS		Bayesian	
β	δ	n		Bias	RMSE	Bias	RMSE	Bias	RMSE
0.5	0.5	50	α	0.0763	0.4108	-0.0188	0.1031	0.0037	0.0208
			β	-0.0087	0.4392	0.0655	0.1490	0.0605	0.1722
			δ	-0.0024	0.0589	0.0063	0.0034	0.0116	0.0454
		100	α	0.0250	0.2283	-0.0174	0.0449	0.0036	0.0265
			β	-0.0189	0.2722	0.0337	0.0669	0.0336	0.1440
			δ	-0.0014	0.0390	0.0043	0.0015	0.0067	0.0312
		200	α	0.0238	0.1901	-0.0031	0.0208	0.0048	0.0325
			β	0.0055	0.1994	0.0347	0.0280	0.0299	0.1179
			δ	0.0005	0.0268	0.0040	0.0007	0.0047	0.0231
	2	50	α	0.0848	0.4883	0.0110	0.1802	0.0040	0.0220
			β	-0.0053	0.5195	0.0884	0.2352	0.0557	0.1710
			δ	-0.0185	0.2605	0.0229	0.0624	0.0372	0.1659
		100	α	0.0360	0.3164	0.0031	0.0869	0.0022	0.0273
			β	0.0009	0.3449	0.0647	0.1137	0.0381	0.1486
			δ	-0.0052	0.1806	0.0222	0.0309	0.0235	0.1232
		200	α	0.0119	0.2015	-0.0014	0.0383	0.0015	0.0326
			β	-0.0078	0.2116	0.0351	0.0445	0.0183	0.1150
			δ	-0.0067	0.1138	0.0116	0.0126	0.0094	0.0881
2	0.5	50	α	0.0933	0.4091	-0.0698	0.1299	0.0010	0.0210
			β	0.0935	0.8778	0.0208	0.4735	0.0448	0.1068
			δ	0.0078	0.0917	0.0005	0.0060	0.0190	0.0621
		100	α	0.0665	0.3711	-0.0348	0.0763	0.0024	0.0265
			β	0.0732	0.7833	0.0243	0.3001	0.0392	0.1178
			δ	0.0027	0.0625	-0.0007	0.0033	0.0101	0.0416
		200	α	0.0450	0.2147	-0.0192	0.0342	0.0032	0.0295
			β	0.0751	0.4542	0.0291	0.1509	0.0410	0.1219
			δ	0.0045	0.0433	0.0006	0.0017	0.0066	0.0293
	2	50	α	0.3318	0.9574	-0.0362	0.1857	0.0013	0.0214
			β	0.7134	2.4516	0.1022	0.7926	0.0379	0.1031
			δ	0.1614	0.6438	0.0328	0.1295	0.0745	0.2226
		100	α	0.1235	0.4749	-0.0302	0.0936	0.0025	0.0257
			β	0.2302	1.1271	0.0382	0.4345	0.0353	0.1108
			δ	0.0471	0.3434	-0.0017	0.0659	0.0369	0.1543
		200	α	0.0427	0.2836	-0.0292	0.0532	0.0020	0.0291
			β	0.0653	0.6103	-0.0026	0.2270	0.0297	0.1154
			δ	0.0138	0.2215	-0.0075	0.0362	0.0270	0.1238

6 Applications to COVID-19 data

In this section, three real data of COVID-19 from Italy, Mexico and Netherlands are given to test the goodness of the EOWR distribution. The EOWR model is compared with other related models such as, Rayleigh, Marshall-Olkin Rayleigh (MOR) [26], Kumaraswamy exponentiated Rayleigh (KER) Rashwan [27] and extended odd Weibull exponential (EOWE) distribution [15]. Tables 3 4 and 5 provide values of Crammer-von Mises (W*), Anderson-Darling (A*) and Kolmogorov- Smirnov (KS) statistic along with its

P-value for all models fitted based on three real data sets.

The first data represents a COVID-19 data belong to Italy of 59 days, that is recorded from 27 February to 27 April 2020. This data formed of rough mortality rate. The data are as follows: 4.571 7.201 3.606 8.479 11.410 8.961 10.919 10.908 6.503 18.474 11.010 17.337 16.561 13.226 15.137 8.697 15.787 13.333 11.822 14.242 11.273 14.330 16.046 11.950 10.282 11.775 10.138 9.037 12.396 10.644 8.646 8.905 8.906 7.407 7.445 7.214 6.194 4.640 5.452 5.073 4.416 4.859 4.408 4.639 3.148 4.040 4.253 4.011 3.564 3.827 3.134 2.780 2.881 3.341 2.686 2.814 2.508 2.450 1.518.

Table 3: MLE estimates, SE, KS test, P-values, W* and A* for COVID-19 data of Italy

Italy	α	β	θ	λ	W*	A*	KS	P-value
EOWR	2.9019	15.8688	0.0551		0.0653	0.3685	0.0828	0.7819
	1.0417	9.3609	0.0170					
R	6.5829				0.1328	0.8044	0.1360	0.2056
	0.4285							
MOR	0.7578	7.0444			0.1343	0.7990	0.1137	0.4004
	0.3509	0.9770						
KER	0.0131	1.6751	1.8468	0.5114	0.1325	0.8022	0.1219	0.3184
	0.0172	2.1649	3.1031	0.8593				
EOWE	1.4103	0.0356	0.0710		0.1324	0.8023	0.1239	0.3004
	0.2710	0.1753	0.0116					

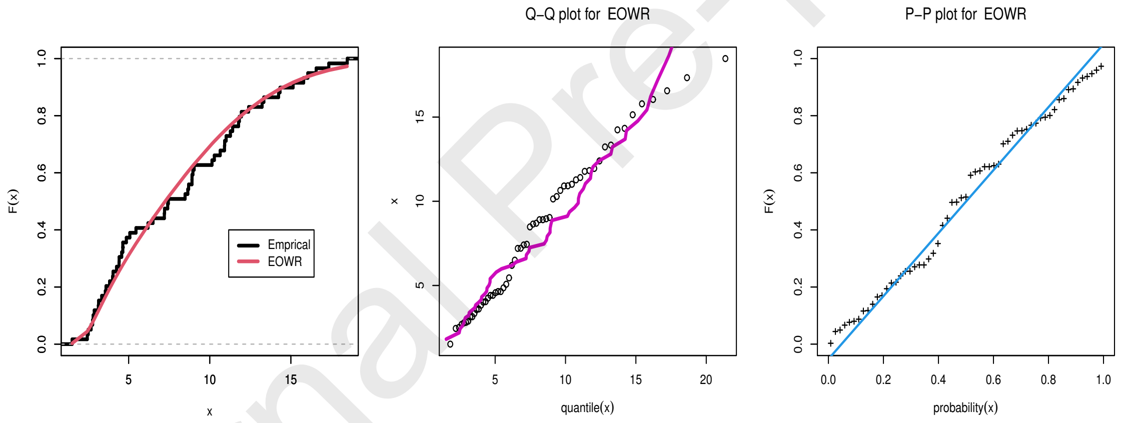

Figure 3: Estimated PDF, PP-plot and QQ-plot of EOWR for COVID-19 data of Ital.

The second data represents a COVID-19 data belong to Mexico of 108 days, that is recorded from 4 March to 20 July 2020. This data formed of rough mortality rate. The data are as follows: 8.826 6.105 10.383 7.267 13.220 6.015 10.855 6.122 10.685 10.035 5.242 7.630 14.604 7.903 6.327 9.391 14.962 4.730 3.215 16.498 11.665 9.284 12.878 6.656 3.440 5.854 8.813 10.043 7.260 5.985 4.424 4.344 5.143 9.935 7.840 9.550 6.968 6.370 3.537 3.286 10.158 8.108 6.697 7.151 6.560 2.988 3.336 6.814 8.325 7.854 8.551 3.228 3.499 3.751 7.486 6.625 6.140 4.909 4.661 1.867 2.838 5.392 12.042 8.696 6.412 3.395 1.815 3.327 5.406 6.182 4.949 4.089 3.359 2.070 3.298 5.317 5.442 4.557 4.292 2.500 6.535 4.648 4.697 5.459 4.120 3.922 3.219 1.402 2.438 3.257 3.632 3.233 3.027 2.352 1.205 2.077 3.778 3.218 2.926 2.601 2.065 1.041 1.800 3.029 2.058 2.326 2.506 1.923.

Table 4: MLE estimates, SE, KS test, P-values, W* and A* for COVID-19 data of Mexico

Mexico	α	β	θ	λ	W*	A*	KS	P-value
EOWR	1.9711	6.6509	0.0633		0.0293	0.1777	0.0449	0.9815
	0.5438	3.8516	0.0227					
R	4.6719				0.1185	0.7626	0.0934	0.3027
	0.2248							
MOR	0.6115	5.3157			0.0822	0.5155	0.0602	0.8283
	0.2121	0.6052						
KER	0.2670	0.1734	0.9477	1.3805	0.0955	0.6087	0.1227	0.0773
	0.0021	0.0169	0.0663	0.0680				
EOWE	2.1998	1.1979	0.1406		0.0908	0.5128	0.0736	0.6017
	0.4146	0.6327	0.0180					

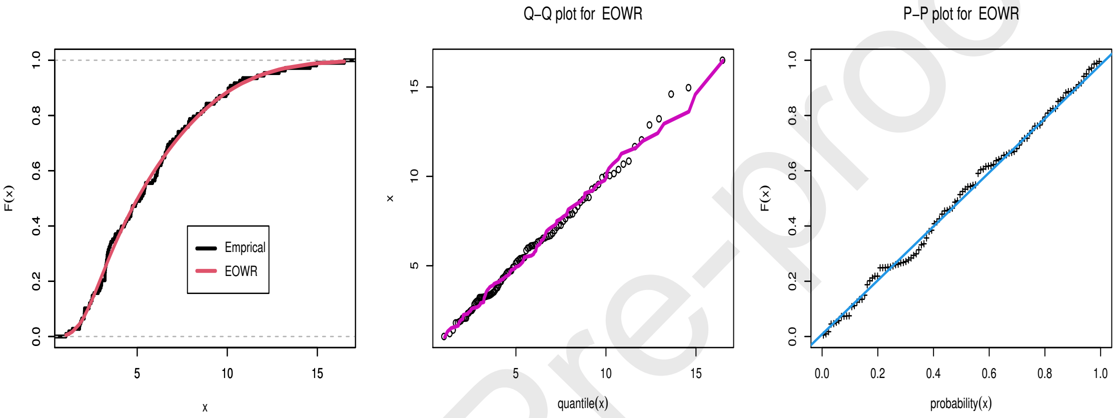

Figure 4: Estimated PDF, PP-plot and QQ-plot of EOWR for COVID-19 data of Mexico.

The third data represents a COVID-19 data belong to Netherlands of 30 days, that is recorded from 31 March to 30 April 2020. This data formed of rough mortality rate. The data are as follows: 14.918 10.656 12.274 10.289 10.832 7.099 5.928 13.211 7.968 7.584 5.555 6.027 4.097 3.611 4.960 7.498 6.940 5.307 5.048 2.857 2.254 5.431 4.462 3.883 3.461 3.647 1.974 1.273 1.416 4.235.

Table 5: MLE estimates, SE, KS test, P-values, W* and A* for COVID-19 data of Netherlands

Netherlands	α	β	δ	λ	W*	A*	KS	P-value
EOWR	1.3172	2.7624	0.0335		0.0262	0.1807	0.0734	0.9932
	0.4285	2.2505	0.0170					
R	4.9985				0.0520	0.3158	0.1167	0.7655
	0.4563							
MOR	0.6136	5.6754			0.0359	0.2301	0.0832	0.9746
	0.3955	1.1974						
KER	0.0115	3.2253	2.3294	0.4031	0.0506	0.3084	0.1046	0.8646
	0.0203	5.5682	7.5032	1.2984				
EOWE	2.0538	1.0192	0.1274		0.0272	0.1873	0.0827	0.9758
	0.6509	0.9289	0.0267					

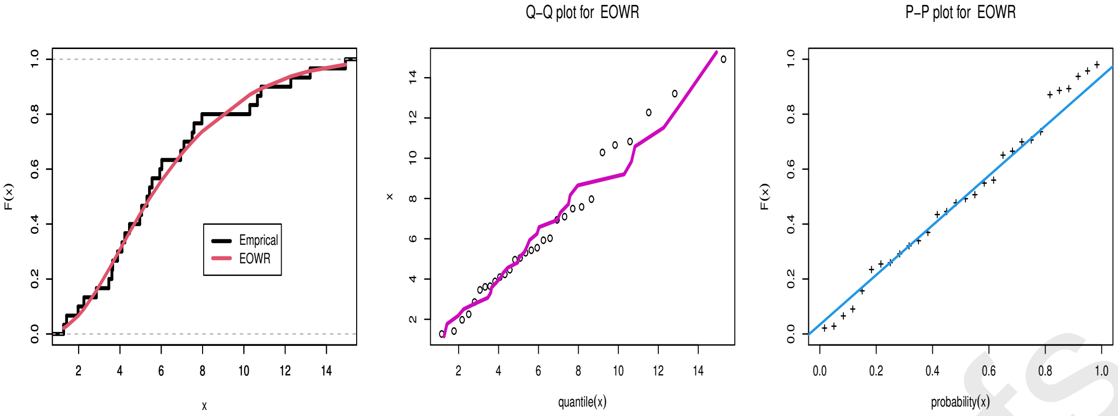

Figure 5: Estimated PDF, PP-plot and QQ-plot of EOWR for COVID-19 data of Netherlands.

From Tables 3 4 and 5 it is obvious that EOWR distribution has minimum values of all information criteria compared with other distributions. Also the P-value for KS has its highest value when the life time is EOWR distribution. This leads us to conclude that EOWR better fit the three real sets of data. Empirical, Q-Q and P-P plots shown in figures 3, 4 and 5, indicate that our distribution is a good choice for modeling the above real data.

7 Conclusion

In this paper we formulate a new generalization of Rayleigh and Weibull distributions which is called EOWR distribution. We studied its statistical properties and obtained a linear representation for its pdf which was efficient in finding moments, moment generating function, mean residual and others. Different classical and bayes estimation methods were considered to find point estimation of EOWR unknown parameters α, β and δ. A comparison was conducted via simulation analysis using R package to distinguish the performance of different estimation method. MCMC method was used for that purpose, also real data sets were considered and they showed that EOWR better fit these data compared with other competitive distributions.

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